PRACTICAL MATHEMATICS

PRACTICAL MATHEMATICS

FOR STUDENTS ATTENDING EVENING,
DAY CONTINUATION AND JUNIOR
TECHNICAL SCHOOLS

BY

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SECOND EDITION

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PREFACE TO THE FIRST EDITION

At the present time there are many Teachers of Mathematics in Day and Evening Technical Schools who require examples suitable for Engineering Students. The accompanying volume attempts to provide a collection of such examples. It may be used by Teachers whose knowledge is non-technical, since explanations of technical terms, etc., are given wherever necessary. At the same time it has been written specially for the student; for in each chapter there are *numerous examples fully worked out,* and at the end of each chapter there are examples of a *similar nature* to be worked out by the student himself.

Some of the examples are easy, others are difficult. Those which are difficult ought to be left for a second or a third reading. A portion of section 2, chapter II., must be left for a second reading, because it involves methods which are given later. The last chapter on "Transformation of Formulae" ought to be referred to very frequently from the beginning onwards, because the manipulation of formulae is of great importance to engineers. An "Appendix," containing notes on "Interpolation" and "Taper," has been added, and will be found useful. Reference is often made to it, the references being stated in the text.

The aim of the book is to treat those Plane and Solid Figures with which engineers are most familiar, in such a manner that a student may make calculations on the appliances he sees and uses in daily life. This necessitates the use of Algebra and Trigonometry in a practical way, *i.e.* in the applied sense. In this way a student may cultivate the habit of observing carefully what he sees in the workshops and elsewhere.

The author's experience goes to show that *logarithms* should be used from the beginning.* Calculations are simplified, time is saved and labour lessened.

The author wishes to convey his thanks to several friends for reading over the MS., also to some of his former students for working out the answers to the examples at the ends of the chapters.

N. W. M'LACHLAN.

NEWCASTLE, 1913.

PREFACE TO THE SECOND EDITION

THIS is not a text book in the usual sense. The text is replaced by a large number of worked examples drawn from various phases of engineering work and elsewhere. The prefatory notes to each chapter are inserted merely for reference.

In using the book it is intended that the Lecturer will provide the necessary explanation of the underlying principles, and will use his discretion regarding the sequence in which the various chapters are studied.

There is a distinct co-ordination between graphical and analytical work throughout the book. In many cases problems are solved graphically by drawing to scale, but analytical methods are used as a check and *vice-versa*. The student is thus able to study elementary mathematical analysis and methods of graphical treatment, which are so beneficial in the drawing office and in all engineering work.

The book covers a large portion of the mathematical course in Junior Technical Schools. It also covers part of the work of Day Continuation Schools for apprentices in Engineering and other Trades.

N. W. M'LACHLAN.

LONDON, 1920.

* See p. 170 for method of arranging logarithmic computations.

CONTENTS

ABBREVIATIONS.

$=$	means	"equal to."	$\neq$	means	"not equal to."
$>$	,,	"greater than."	$<$	,,	"less than."
$\ngtr$	,,	"not greater than."	$\nless$	,,	"not less than."
$\hat{ABC}$	,,	"angle ABC."	$\hat{A}$	,,	"angle A."
ins.	,,	"inches."	ft.	,,	"feet."
yds.	,,	"yards."	mm.	,,	"millimetres."
cms.	,,	"centimetres."	dm.	,,	"decimetres."
sq. ins.	,,	"square inches."	cu. ins.	,,	"cubic inches."
16° 15′	,,	"16 degrees, 15 minutes."			
3^c	,,	"3 radians."			
r.p.s.	,,	"revolutions per second."			
r.p.m.	,,	" ,, minute."			
m.p.h.	,,	"miles per hour."			
dia.	,,	"diameter."			
rad.	,,	"radius" and sometimes "radians."			
lbs.	,,	"pounds."			
H.P.	,,	"horizontal plane" and sometimes "horse-power."			

CONSTANTS.

$\pi = \frac{22}{7}$, 3·1416, 3·142 approximately.*

$\log \pi = 0·4971499$, which may be taken correct to 4 decimal places, as 0·4971 or 0·4972. The value 0·4972 has been used for calculations in this book.

$\frac{1}{\pi} = 0·3183.$ $\frac{\pi}{4} = 0·7854.$ $\frac{\pi}{6} = 0·5236.$

$\pi^2 = 9·87$ or 10 approximately.

2π radians = 360 degrees.

1 radian = 57·3 degrees approximately.

1 inch = 2·54 centimetres.

1 cubic foot of fresh water	weighs	62·5 lbs. = 6·25 gallons.
1 gallon of fresh water	,,	10 lbs.
1 square foot of ¼″ steel plate	,,	5·1 lbs.
1 cubic inch of cast iron	,,	0·26 lb.
1 ,, ,, steel	,,	0·28 lb.
1 ,, ,, steel plate	,,	0·283 lb.

1 naut = 1 nautical mile = 6080 feet.

1 knot = 1 nautical mile per hour = 6080 feet per hour.

* π is an incommensurable quantity, i.e. it cannot be determined exactly.

PRACTICAL MATHEMATICS

CHAPTER I.

PLANE RECTILINEAR FIGURES

The Rectangle. A rectangle is a plane 4-sided rectilinear * figure with its opposite sides parallel and all its angles right angles (Fig. 1).

AB = DC,
AD = BC
and AC = BD.

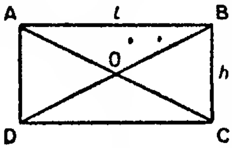


Fig. 1.

AC and BD are diagonals. O is the centre of the rectangle. The triangles AOB, BOC, COD, DOA are all equal in area. The diagonals bisect each other.

$$\text{Area} = AB \times BC = \underline{lh}.$$

$$\text{Perimeter or sum of sides} = 2l + 2h = \underline{2(l+h)}.$$

The Square. When the length of a rectangle is equal to the height, the rectangle is a square (Fig. 2).

$$\text{Area} = AB \times BC = \underline{a^2}.$$
$$\text{Perimeter} = \underline{4a}.$$

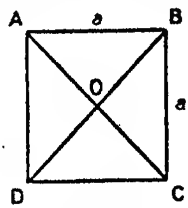


Fig. 2.

* A figure bounded solely by straight lines.

The Parallelogram. A parallelogram (Fig. 3) is a plane 4-sided rectilinear figure with its opposite sides parallel and none of its angles right angles. When the angles are right angles the figure is a rectangle; and if its sides are equal too it is a square.

$AB = DC$,

$AD = BC$,

$\hat{A} = \hat{C}$

and $\hat{B} = \hat{D}$.

The four triangles are equal in area.

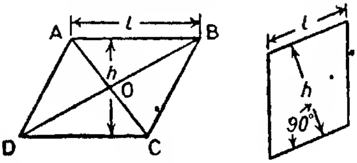


Fig. 3

$$\text{Area} = \text{length of base} \times \text{altitude}$$
$$= \underline{lh}.$$

Consequently, if two parallelograms are on the same base and have the same altitude, *i.e.* are between the same parallels, they are equal in area.

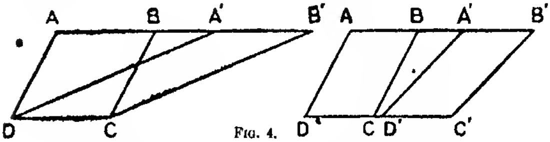


Fig. 4.

$ABCD = A'B'CD$. $\qquad ABCD = A'B'C'D'$.

Also, if two parallelograms have equal bases and are between the same parallels, they are equal in area; for this may be shown by assuming $A'B'CD$ to slide between parallel guides and take the position $A'B'C'D'$ (Fig. 4).

$$\text{Perimeter} = 2l + 2s$$
$$= \underline{2(l+s)}.$$

Now, $\dfrac{h}{s} = \sin\alpha$, $\therefore\ h = s\sin\alpha$ (see p. 33).

$$\therefore\ s = h/\sin\alpha.$$

$$\therefore\ \text{perimeter} = \underline{2\left(l + \frac{h}{\sin\alpha}\right)}.$$

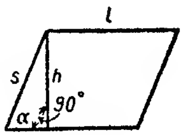


Fig. 5.

$$\text{Area} = lh$$
$$= \underline{ls\sin\alpha}.$$

The Rhombus. When the sides of a parallelogram are all equal and none of the angles right angles, the figure is a rhombus (Figs. 6 and 7).

$$\text{Area} = lh.$$
$$= l^2 \sin \alpha \text{ (see page 33).}$$
$$\text{Perimeter} = 2(l+l) = 4l.$$

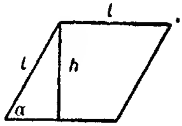


FIG. 6.

The diagonals AC, BD bisect each other at 90°. Hence the area of

$$\triangle ADB = \frac{AC}{2} \times \frac{BD}{2} \text{ (see page 5),}$$

and area of

$$\triangle BDC = \frac{AC}{2} \times \frac{BD}{2}.$$

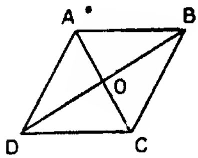


FIG. 7.

$$\therefore \text{ total area } ABCD = \frac{AC \cdot BD}{4} + \frac{AC \cdot BD}{4}$$
$$= \frac{AC \cdot BD}{2}$$
$$= \frac{d_1 d_2}{2},$$

where d_1 and d_2 are the lengths of the diagonals.

The Quadrilateral. A quadrilateral is a plane 4-sided rectilinear figure (Fig. 8).

$$\text{Area} = \triangle ABC + \triangle ADC$$
$$= \frac{AC}{2} \times p_2 + \frac{AC}{2} \times p_1 \text{ (see page 5)}$$
$$= \frac{AC}{2}(p_1 + p_2).$$

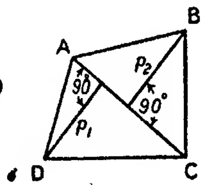


FIG. 8.

p_1 and p_2, the perpendiculars on the diagonal AC, are found by a scale drawing. Either diagonal may be chosen.

$p_1 = \mathrm{OD}\sin\alpha$ (see p. 33),

$p_2 = \mathrm{OB}\sin\alpha.$

$\therefore (p_1 + p_2)$

$= (\mathrm{OD} + \mathrm{OB})\sin\alpha$

$= \mathrm{DB}\sin\alpha.$

$\therefore \frac{\mathrm{AC}}{2}(p_1 + p_2)$

$= \frac{\mathrm{AC}}{2}.\mathrm{DB}.\sin\alpha.$

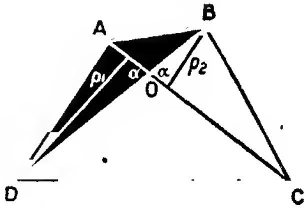


FIG. 9.

$$\therefore \text{area} = \frac{d_1 d_2 \sin\alpha}{2}.$$

Referring to the rhombus (Fig. 6), $\alpha = 90°$, $\therefore \sin\alpha = 1$, and the

$$\text{Area} = \frac{d_1 d_2}{2}, \text{ as there shown.}$$

The above method is very convenient when a protractor is available. In general it is better to measure the acute angle between the diagonals, because it is simpler to deal with.

The perimeter = AB + BC + CD + DA.

Rectangles, squares, etc., are merely particular cases of the quadrilateral.

The Triangle. The triangle can be shown to be a particular case of the quadrilateral when one side becomes indefinitely short. Thus, in Fig. 9, if AB becomes infinitely small, the resulting figure is a triangle ADC.

A triangle may be defined as a plane three-sided rectilinear figure (Fig. 10).

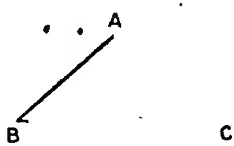


FIG. 10.

There are three kinds of triangle:

(1) Equilateral, having three equal sides.

(2) Isosceles, „ two „ „

(3) Scalene, „ three unequal sides.

Further sub-divisions result in :

(1) Right-angled triangles, having one angle 90°.
(2) Acute-angled " " three angles $< 90°$
(3) Obtuse-angled " " one angle $> 90°$.

In cases (1) and (3) the triangles may be isosceles or scalene. In case (2) they may be equilateral, isosceles or scalene.

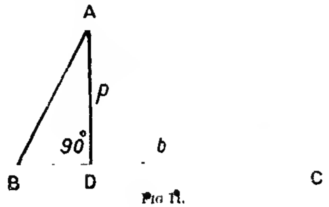


FIG. 11.

Let ABC (Fig. 11) be *any* triangle, then we have the following:

(1) The sum of any two sides > the third side,

e.g. $AB + BC > AC$.

(2) $\hat{A} + \hat{B} + \hat{C} = 180°$.

(3) The area $= \dfrac{AD \times BC}{2} = \dfrac{pb}{2}$, *i.e.* half the area of a parallelogram on the same base and having the same altitude.

EXAMPLES OF TRIANGLES.

(*a*) **Equilateral.**

$\hat{A} = \hat{B} = \hat{C} = 60°$,

$AB = BC = CA = a$.

Note.—Area $= \dfrac{\sqrt{3}}{4} a^2 = 0{\cdot}433a^2$.

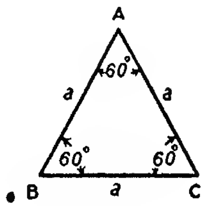


FIG. 12.

(*b*) **Isosceles.**

$\hat{B} = \hat{C}$ (the angles at the base),

$AB = AC$.

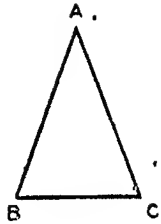


Fig. 13.

(*c*) **Scalene.**

$\hat{A} \neq \hat{B} \neq \hat{C}$,

$AB \neq BC \neq CA$.

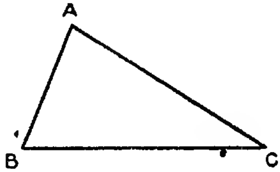


Fig. 14.

(*d*) **Right-angled (Scalene).**

$\hat{C} = 90^\circ$,

$\hat{A} + \hat{B} = 90^\circ$,

$\hat{A} \neq \hat{B}$,

$AB > AC$, $AB > BC$, $AC \neq BC$. $\left\{\begin{array}{l}\text{AB is the hypotenuse or greatest}\\ \text{side. It is opposite to the right angle.}\end{array}\right\}$

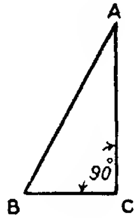


Fig. 15.

(*e*) **Right-angled (Isosceles).**

$\hat{C} = 90^\circ$,

$\hat{A} = \hat{B} = 45^\circ$,

$AB > BC$, $AB > AC$,

$BC = AC$.

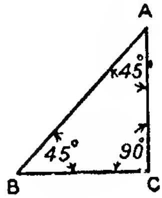


Fig. 16.

(*f*) **Acute-angled.**

$\hat{A} < 90^\circ$,
$B < 90^\circ$,
$C < 90^\circ$.

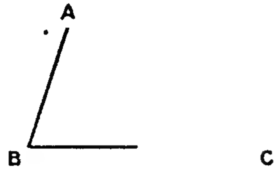


Fig. 17.

(*g*) **Obtuse-angled.**

$\hat{A} > 90^\circ$,
$B < 90^\circ$,
$C < 90^\circ$.

Fig. 18.

The restrictions imposed are stated in each case. In (*f*), for example, there are three restrictions, *i.e.* each angle must be less than 90°; hence the triangle may be equilateral, isosceles or scalene. The other cases may be treated in a similar manner.

The Trapezium. A trapezium is a plane 4-sided rectilinear figure with one pair of opposite sides parallel (Fig. 19).

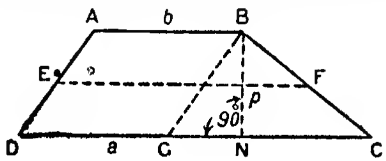


Fig. 19.

$$\text{Area} = \left(\frac{DC + AB}{2}\right) BN$$
$$= \left(\frac{a+b}{2}\right) p$$
$$= \tfrac{1}{2} \text{ sum of parallel sides} \times \text{altitude.}$$

This may be proved as follows:

Draw BG parallel to AD.

$$\text{Area} = \text{parallelogram ABGD} + \text{triangle BGC}$$

$$= \text{DG} \times \text{BN} + \frac{\text{GC} \times \text{BN}}{2}$$

$$= \frac{2\text{DG} \times \text{BN} + \text{GC} \times \text{BN}}{2}$$

$$= \frac{(\text{GC} + \text{DG} + \text{DG})\,\text{BN}}{2}$$

$$= \left(\frac{a+b}{2}\right)p, \text{ because GC} + \text{DG} = a, \text{ and DG} = b.$$

Since $\left(\frac{a+b}{2}\right)$ is the mean of a and b, the

Area = mean of parallel sides × altitude

= EF × altitude, where E and F are the middle points of the non-parallel sides.

EXAMPLES.

1. The length of a rectangle is 85·24 ins. and the height 34·75 ins. Find its area and perimeter.

Area $= lh$
$= 85{\cdot}24 \times 34{\cdot}75$
$= 2961$ sq. ins.

Perimeter $= 2(l+h)$
$= 2(85{\cdot}24 + 34{\cdot}75)$
$= 2 \times 119{\cdot}99$
$= 240$ ins. about.

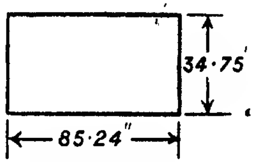


FIG. 20.

2. Find the weight of the above if it was a steel plate $\frac{5}{8}$ in. thick. 1 sq. ft. $\frac{1}{8}$ in. steel plate weighs 5·1 lbs.

$$\text{Weight} = \text{area in sq. feet} \times \text{wt. of 1 sq. ft. of } \tfrac{5}{8} \text{ in. plate}$$
$$= \frac{2961 \times 5{\cdot}1 \times 5}{144}$$
$$= \frac{2961 \times 25{\cdot}5}{144}$$
$$= \underline{524{\cdot}3 \text{ lbs., say } 524 \text{ lbs.}}$$

Notice that $5{\cdot}1 \times 5$ is the weight of 1 sq. ft. of $\frac{5}{8}$ in. plate, because 1 sq. ft. of $\frac{5}{8}$ in. plate is five times as heavy as 1 sq. ft. of $\frac{1}{8}$ in. plate.

3. Find the area of the doubly symmetrical I section shown.

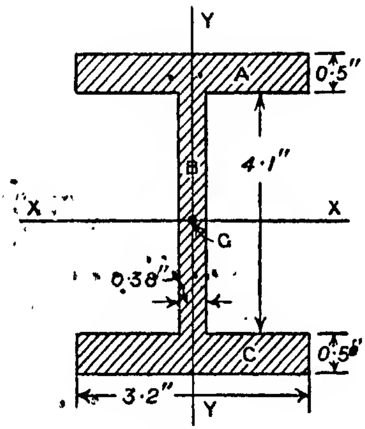


FIG. 21.

$$\text{Area of section} = A + C + B$$
$$= 2A + B, \text{ since it is symmetrical,}$$
$$= 2 \times 3{\cdot}2 \times 0{\cdot}5 + 0{\cdot}38 \times 4{\cdot}1$$
$$= 3{\cdot}2 + 1{\cdot}56$$
$$= \underline{4{\cdot}76 \text{ sq. ins.}}$$

The above section (Fig. 21) comes under the doubly symmetrical classification, because it is the same on both sides of

the two rectangular axes XX, YY, passing through its centre of gravity G. If a section is the same on both sides of one rectangular axis it is singly symmetrical.

4. The area of a rectangle is 981 sq. cms. and the length is $\frac{1}{3}$ the height. Find its dimensions.

Area $= lh$;

but $l = h/3$.

$\therefore A = \frac{h^2}{3}$;

$\therefore \frac{h^2}{3} = 981$;

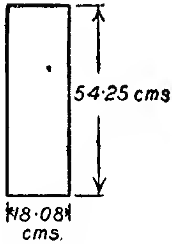


FIG. 22.

$\therefore h^2 = 981 \times 3 = 2943$;

$\therefore h = \sqrt{2943}$

$= \underline{54{\cdot}25 \text{ cms.}}$

$l = h/3$

$= \frac{54{\cdot}25}{3}$

$= \underline{18{\cdot}08 \text{ cms.}}$

5. The plan of a slide valve is 10·5 ins. × 7·75 ins. and the resultant pressure on the back 85 lbs. per sq. in. Find the total force pressing the valve on the port facings.

Area of plan $= 10{\cdot}5 \times 7{\cdot}75$ sq. ins.

Pressure per square inch = 85 lbs.

Hence total pressure $= 10{\cdot}5 \times 7{\cdot}75 \times 85$

$= \underline{6916 \text{ lbs., say } 6920 \text{ lbs.,}}$

which is a little more than 3 tons.

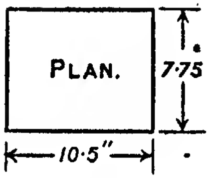


FIG. 23.

6. A square has an area of 78·2 sq. ft.; find the length of a side and a diagonal.

Area $= a^2$;

$\therefore\ a^2 = 78{\cdot}2$;

$\therefore\ a = \sqrt{78{\cdot}2}$

$= \underline{8{\cdot}842 \text{ ft.}}$

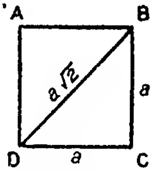


FIG. 24.

$\text{DB}^2 = \text{DC}^2 + \text{BC}^2$, since $\hat{\text{C}}$ is 90° (see page 23);

$\therefore\ \text{DB}^2 = a^2 + a^2 = 2a^2$;

$\therefore\ \text{DB} = a\sqrt{2}$.

Hence $\text{DB} = 8{\cdot}842 \times \sqrt{2}$

$= 8{\cdot}842 \times 1{\cdot}414$

$= \underline{12{\cdot}52 \text{ ft.}}$

7. The base of a parallelogram is 10·2″ and the altitude 7·45″. Find the area.

Area = base × altitude

$= 10{\cdot}2 \times 7{\cdot}45$

$= \underline{76 \text{ sq. ins.}}$

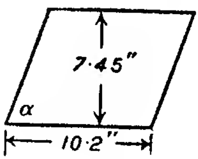


FIG. 25.

Notice that the angle α is not stated.

This is immaterial, since all parallelograms on the same base and *between the same parallels are equal in area.*

8. The base of a parallelogram is 10 ins. and the angle $\alpha = 50°$. Find the area if the slant side $s = 6$ ins.

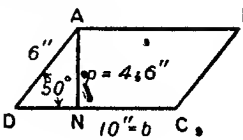


FIG. 26.

Set off DC = 10 ins. to scale.
Draw DA at 50° to DC = 6 ins. to scale.
Draw CB equal and parallel to DA.
Join AB.
Draw AN perpendicular to DC.
Measure AN.

Then $$\text{area} = bp$$
$$= 10 \times 4{\cdot}6$$
$$= \underline{46 \text{ sq. ins.}}$$

Or $$\frac{p}{\text{AD}} = \sin 50° \text{ (see page 32)};$$
$$\therefore\ p = \text{AD} \sin 50°$$
$$= 6 \times 0{\cdot}766$$
$$= \underline{4{\cdot}596 \text{ ins.}}$$

Hence $$\text{area} = 10 \times 4{\cdot}596$$
$$= \underline{45{\cdot}96 \text{ sq. ins.}}$$

The above result, viz. 46 sq. ins., is quite near enough for all practical purposes.

9. Find the area of and solve the following triangle by drawing to scale:

$$\hat{A} = 100°, \quad \hat{B} = 54°, \quad a = 16 \text{ ins.}$$

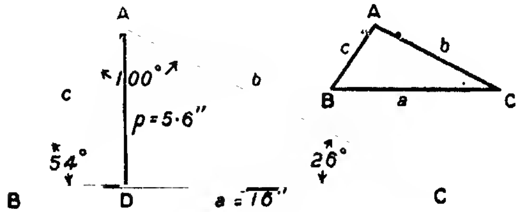


FIG. 27.

Set off BC to scale = 16 ins.
Draw a line BA at 54° to BC.

Since $\hat{C}$ is yet unknown, AC cannot be drawn.

$$\text{But } \hat{A}+\hat{B}+\hat{C}=180^\circ;$$

$$\therefore \hat{C}=180-\hat{A}-\hat{B}$$
$$=180^\circ-100-54$$
$$=180^\circ-154^\circ$$
$$=\underline{26^\circ}.$$

AC can now be drawn, by making $\hat{C}=26^\circ$.
Draw AD perpendicular to BC. Measure AD.

Then $$\text{area}=\frac{\text{base}\times\text{perpendicular to base}}{2},$$

$$=\frac{ap}{2}$$

$$=\frac{\overset{8}{\cancel{16}}\times 5\cdot6}{\cancel{2}}$$

$$=\underline{44\cdot8 \text{ sq. ins.}}$$

To solve a triangle is to determine all its sides and angles. Hence measure the sides c and b.

The solution is: $\begin{cases} a=16 \text{ ins.,} \\ b=13\cdot1 \text{ ins.,} \\ c=7\cdot1 \text{ ins.,} \\ \hat{A}=100^\circ, \\ \hat{B}=54^\circ, \\ \hat{C}=26^\circ. \end{cases}$

Always state the scale to which a figure is drawn before commencing to draw it. When drawings have to be checked—as is the case in all factories—the scale is absolutely indispensable. If too small a scale is chosen, considerable inaccuracies ensue due to measurement. The percentage error is smaller when a large figure is drawn, because one is liable to make the same error in measuring 2 ins. as in measuring 4 ins. In this case the percentage error in the length is halved.

10. The angle of a triangular notch is 80°, and the water level is 6 ins. above the vertex. Find the sectional area of the water.

Area of section $= 6b$.

Now $\frac{b}{6} = \tan 40°$ (see page 32);

$\therefore\ b = 6 \tan 40°$

$= 6 \times 0{\cdot}8391.$

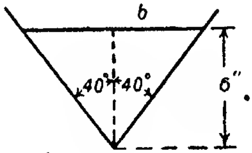


FIG. 28.

By substitution $\quad A = 6 \times 6 \times 0{\cdot}8391$

$= 36 \times 0{\cdot}8391$

$= \underline{30{\cdot}21 \text{ sq. ins.}}$

This problem may also be solved by finding b from a scale drawing.

11. Find the area of the given quadrilateral (use two methods) by drawing to scale:

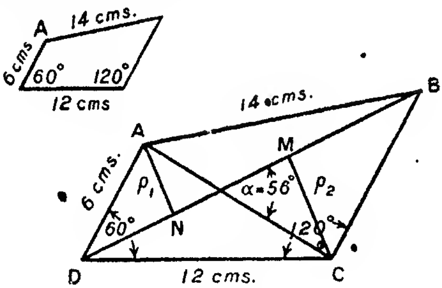


FIG. 29.

Set off DC = 12 cms. to scale.

Draw AD = 6 cms. to scale at 60° to DC.

Draw CB at 120° to DC.

Take 14 cms. to scale on compasses, and with centre A draw an arc cutting CB in B. Join AB.

Draw the diagonals DB and AC.

From A and C draw perpendiculars p_1 and p_2 to the diagonal.

Measure DB, p_1 and p_2.

(*a*)
$$\text{Area} = \left(\frac{p_1+p_2}{2}\right)\text{DB}$$
$$= \frac{(3{\cdot}5+5{\cdot}25)}{2} \times 18{\cdot}5$$
$$= 8{\cdot}75 \times 9{\cdot}25$$
$$= \underline{80{\cdot}94 \text{ sq. cms.}}$$

(*b*) Measure the angle α.

Obtain $\sin \alpha$ from tables.

Then
$$\text{area} = \frac{d_1 d_2}{2} \sin \alpha$$
$$= 18{\cdot}5 \times 10{\cdot}5 \times \sin 56°$$
$$= 18{\cdot}5 \times 10{\cdot}5 \times 0{\cdot}829$$
$$= \underline{80{\cdot}55 \text{ sq. cms.}}$$

Notice the slight difference between the results. This is of course due to errors in drawing and measurement. Let us say the area is 81 sq. cms.

12. Find the weight of the quadrilateral in the last example if it is $\frac{3}{4}''$ steel plate. 1 sq. ft. of $\frac{1}{8}''$ plate = 5·1 lbs. $1'' = 2{\cdot}54$ cms.

$$\text{Weight} = \text{area in sq. ft.} \times \text{wt. of 1 sq. ft.}$$

Now $\quad 1 \text{ in.} = 2{\cdot}54 \text{ cms.};$

$$\therefore \; 12 \text{ in.} = 12 \times 2{\cdot}54 \text{ cms.};$$

$$\therefore \; 12^2 = 144 \text{ sq. ins. or 1 sq. ft.} = (12 \times 2{\cdot}54)^2 \text{ sq. cms.}$$
$$= 144 \times 6{\cdot}45;$$

$$\therefore \; 1 \text{ sq. cm.} = \frac{1}{144 \times 6{\cdot}45} \text{ sq. ft.};$$

$$\therefore \; \text{weight} = \frac{\overset{27}{\cancel{81}} \times \cancel{5{\cdot}1}^{\,8} \times 6}{\underset{24}{\cancel{144}} \times 6{\cdot}45} \qquad (6 \times \tfrac{1}{8}'' = \tfrac{3}{4}'')$$
$$= \underline{2{\cdot}67 \text{ lbs.}}$$

13. A rhombus 4 ins. side has an angle of 65°. Find (*a*) the area of the rhombus, (*b*) the radius of the inscribed circle.

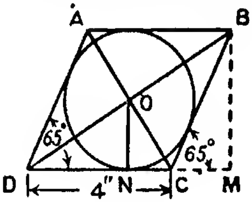


FIG. 30.

Set off DC = 4 ins. to scale.

Draw DA at 65° to CD = 4 ins. to scale.

Draw BC equal and parallel to DA.

Join AB, AC and DB. Draw ON perpendicular to DC. ON is the radius of the inscribed circle.

With O as centre and ON as radius describe a circle. This is the inscribed circle.

Measure ON, AC, DB.

$$\text{Area} = \frac{d_1 \times d_2}{2} \quad (\text{AC} = d_1, \text{DB} = d_2)$$

$$= \frac{4{\cdot}32 \times \cancel{6{\cdot}8}^{\,3{\cdot}4}}{\cancel{2}}$$

$$= \underline{14{\cdot}72 \text{ sq. ins.}}$$

The radius of the inscribed circle may be calculated thus:

$$\text{BM} = 2\text{ON} = 2r.$$

But $\dfrac{\text{BM}}{\text{BC}} = \sin 65°$ (see page 32);

$$\therefore \text{BM} = \text{BC} \sin 65°$$

$$= 4 \times 0{\cdot}9063;$$

$$\therefore \text{ON} = \frac{\text{BM}}{2} = 2 \times 0{\cdot}9063;$$

$$\therefore r = \underline{1{\cdot}813 \text{ ins.}}$$

<u>*r* by measurement was about 1·8 ins.</u>

14. Find the area of the given trapezium by drawing to scale.

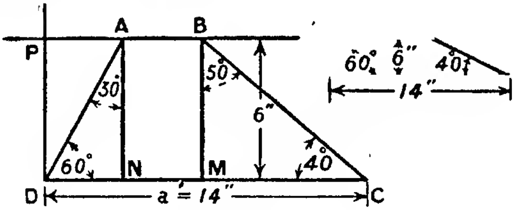


FIG. 31.

Draw DC to scale = 14 ins.

Make $\hat{D} = 60°$ and $\hat{C} = 40°$.

Draw DP perpendicular to DC and make DP = 6 ins.

Draw a line through P parallel to DC, to meet DA and CB in A and B.

Measure AB.

Then
$$\text{area} = \tfrac{1}{2}\text{ sum of parallel sides} \times \text{altitude}$$
$$= \frac{(a+b)}{2}p$$
$$= \frac{(14+3{\cdot}4)}{\cancel{2}} \times \overset{3}{\cancel{6}} \quad (\text{AB} = b = 3{\cdot}4'')$$
$$= 17{\cdot}4 \times 3$$
$$= \underline{52{\cdot}2 \text{ sq. ins.}}$$

The side AB may also be found by calculation thus:

Draw AN and BM perpendicular to DC.

$$\frac{\text{DN}}{\text{AN}} = \tan 30° \text{ (see page 32)};$$
$$\therefore \text{DN} = \text{AN} \tan 30°$$
$$= 6 \times 0{\cdot}5774$$
$$= \underline{3{\cdot}464 \text{ ins.}}$$
$$\frac{\text{CM}}{\text{BM}} = \tan 50°;$$
$$\therefore \text{CM} = \text{BM} \tan 50°$$
$$= 6 \times 1{\cdot}1918$$
$$= \underline{7{\cdot}1508 \text{ ins., say } 7{\cdot}151''.}$$

$$\begin{aligned} AB = NM &= DC - DN - CM \\ &= 14 - 3{\cdot}464 - 7{\cdot}151 \\ &= 14 - 10{\cdot}615 \\ &= \underline{3{\cdot}385 \text{ ins.}} \end{aligned}$$

Examples to be Worked Out.

1. The length of a rectangle is 95·76″ and height 56·27″. Find its area and perimeter, and make a drawing to a scale of $\frac{1}{40}$ full size.

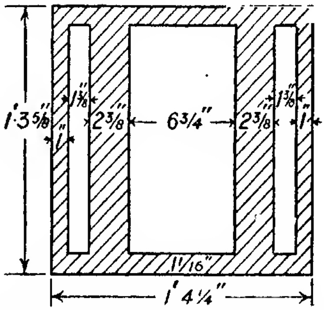
2. The perimeter of a rectangle is 85″ and the ratio of the length to the height 5 : 3·8. Determine its dimensions, and make a suitable scale drawing.

3. The area of a rectangle is 288 sq. ins., and the length is half the height. Find its dimensions. Make a scale drawing.

4. A rectangular piece of steel is 9·75″ × 3·25″ and $\frac{3}{4}$″ thick. Find its weight if 1 sq. ft. of $\frac{1}{8}$″ steel plate weighs 5·1 lbs. Make a scale drawing.

5. The weight of a rectangular steel plate is 15 lbs. and its thickness $\frac{5}{8}$″. If 1 sq. ft. of $\frac{1}{8}$″ steel plate weighs 5·1 lbs., determine its area and dimensions, assuming the length 1·8 times the depth.

6. The figure shows the plan of the lower side of the main slide valve of a steam engine. Find the area of the contact surface (shown shaded) in sq. ins. Make a suitable scale drawing.



7. The plan of a slide valve is 12″ × 8″, and the resultant pressure on the back 75 lbs. per sq. in. Find the total force pressing the valve on the port facing. Draw the plan to scale.

8. An engine cylinder is 65″ diameter, and the stroke 4′ 6″. Calculate the area of its plan when the cylinder is horizontal. Draw the plan to a suitable scale.

9. The floor of a room is 18′ × 15′, and a 2′ 6″ border has to be left all round for varnishing. Find the area of carpet necessary.

10. A room is $19' \times 17' \times 12'$ high. It has a fireplace $6' \times 3'$ and a door $8' \times 3'$. Find the area of wall paper if a border of 12″ is left at the top and 9″ at the bottom.

11. A square bowling green has a side 70·65 yds. long. Find its area.

12. The area of a square is 37·85 sq. mls. Find the length of a side. Make a scale drawing and measure the diagonals.

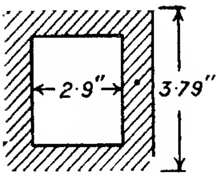
13. A square is inscribed in another square 7·5″ side, so that the diagonals of both are coincident. The area of the inner square is half that of the outer square. Find its dimensions. Make a scale drawing.

14. A rectangular plate whose sides are 2·57 : 3·92 is equal in area to a square plate whose diagonal is 9·72″ long. Find the dimensions of the rectangular plate.

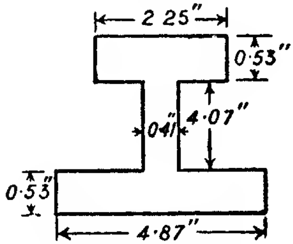
15. The base of a parallelogram is 14″ and the angle $\alpha = 49°$. Find the area if the side s is 10·5″.

16. The base of a parallelogram is 9·75″ and the altitude 7·89″. Find the area. Make a scale drawing.

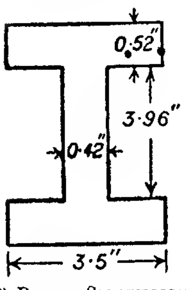
17. Find the areas of the sections shewn. Draw each to scale and shew its axis or axes of symmetry.



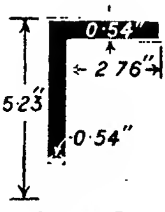
(1) Hollow Square.



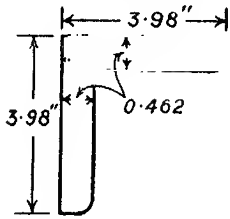
(2) Singly Symmetrical I Section.



(3) Doubly Symmetrical I Section.



(4) Channel Iron.



(5) Angle Iron.

18. Draw a parallelogram $3'' \times 2\frac{1}{2}''$, $\alpha = 68°$. Draw its diagonals, and by measurement shew that they bisect. Also shew that the 4 triangles so formed are all equal in area. Find the area of one of them.

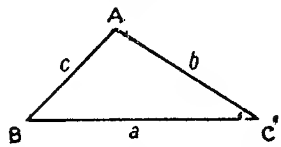
19. Draw a rhombus having a side of $2\frac{1}{2}''$, $\alpha = 57°$. Shew by measurement that its diagonals bisect at 90°. Find its area by two methods.

20. A rhombus has to be constructed equal in area to a parallelogram whose base is $3{\cdot}75''$, slant side $s = 5''$ and $\alpha = 75°$. The angle at the base of the rhombus is 60°. Find the length of its side and construct it.

21. A rhombus has a side 8″ long and an angle of 60°. Find its weight if it is $\frac{5}{8}''$ steel plate. 1 sq. ft. $\frac{1}{8}''$ steel plate $= 5{\cdot}1$ lbs. Make a scale drawing.

22. Find, by calculation and measurement, the distance of the centre of the rhombus in (19) from each side. Hence draw the inscribed circle.

23. Find the areas of and solve the following triangles by drawing to scale:



(1) $a = 2{\cdot}95$ cms., $\hat{B} = 37°$, $\hat{C} = 25°$.

(2) $a = 8{\cdot}97$ ins., $\hat{B} = 58°$, $\hat{A} = 50°$.

(3) $a = 13{\cdot}2$ ft., $\hat{B} = 76°$, $c = 6{\cdot}54$ ft.

(4) $c = 21{\cdot}3$ yds., $\hat{B} = 54°$, $\hat{A} = 100°$.

(5) $c = 10{\cdot}2$ mls., $\hat{A} = 72°$, $\hat{C} = 39°$.

(6) $a = 5$ cms., $b = 6$ cms., $c = 7$ cms.

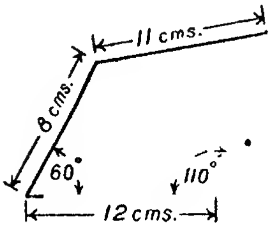
24. A triangular plate is $\frac{1}{2}''$ thick, and has sides $3\frac{1}{2}''$, $4\frac{1}{2}''$ and 6″. Find its weight if 1 sq. ft. of $\frac{1}{8}''$ steel plate $= 5{\cdot}1$ lbs.

25. A triangular steel plate has sides 12″, 16″ and 20″ and weighs 13·75 lbs. Find the thickness. 1 sq. ft. $\frac{1}{8}''$ steel plate = 5·1 lbs. (See Example 3, Chapter IV.)

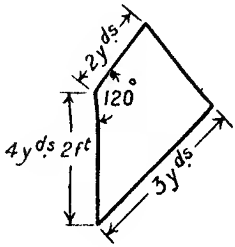
26. A triangular notch has an angle of 90°, and the water level is 7″ above the vertex. Find the area of the section of water.

27. The area of the section of water flowing over a triangular notch is 20 sq. ins. and the height of the water above the apex 5″. Find the angle of the notch by drawing to scale.

28. Find the areas of the following quadrilaterals by two methods. Draw each to scale.

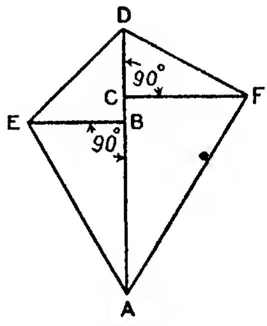


(a)



(b)

AB = 250
AC = 340
AD = 460 } yds.
BE = 195
CF = 230

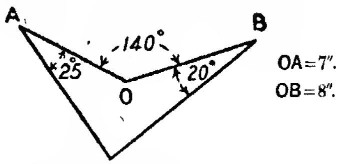


(c)

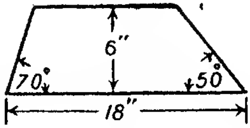
29. Draw a circle 3″ dia. Draw any two diameters at 67° to each other. Join the four circumferential points, and find the area of the quadrilateral so formed. What figure is formed?

30. Find the weight of (a) in (28), assuming it to be $\frac{5}{8}''$ steel plate. 1 sq. ft. $\frac{1}{8}''$ steel plate = 5·1 lbs. 2·54 cms. = 1″.

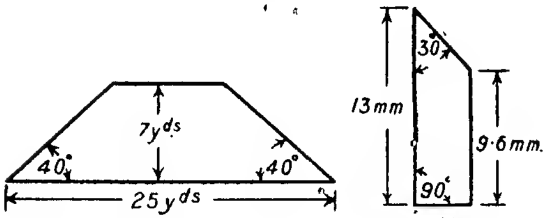
31. Find the area of the given quadrilateral having a re-entrant angle.



32. Find the areas of the following trapezia:



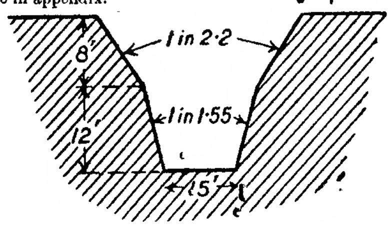
(a)



(b) (c)

33. Find the weight of (a) in (32) if it is $\frac{1}{4}''$ steel plate. 1 sq. ft. $\frac{1}{8}''$ steel plate = 5·1 lbs.

34. Find the area of the cutting shewn. Make a scale drawing. See incline in appendix.



35. Find the area of an equilateral triangle, the length of the sides being 8″ (see page 5).

CHAPTER II.

THE RIGHT-ANGLED TRIANGLE.

SECTION 1.

LET ABC be a right-angled triangle, the angle $\hat{C}$ being 90°. Then $\hat{A}$ and $\hat{B}$ are both acute angles, *i.e.* angles < 90° (Fig. 32).

We have from Euclid, Book I. Proposition 47:

$$AB^2 = AC^2 + BC^2$$

or

$$c^2 = a^2 + b^2 \quad \text{.............................} (\alpha)$$

Stated in words: The square on the hypotenuse is equal to the square on one side + the square on the other side.

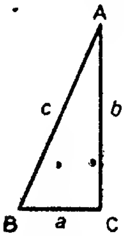


FIG. 32.

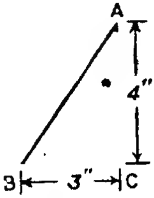


FIG. 33.

Geometrical Representation. (1) Draw, by means of a set square and tee square (or otherwise), a right angle (Fig. 33).

(2) On the vertical leg, mark off 4″ and on the horizontal leg 3″, so that CA = 4″ and CB = 3″. Join AB.

(3) Measure AB, and it will be approximately 5″ long.

Take the equation $AB^2 = AC^2 + BC^2$ and substitute the values from the triangle.

$$AB^2 = AC^2 + BC^2;$$
$$\therefore\ AB^2 = 4^2 + 3^2$$
$$= 16 + 9$$
$$= 25,$$
$$i.e.\quad AB = \sqrt{25} = 5''.$$

Hence, by calculation and drawing $AB = 5''$, which demonstrates the above statement. The relation existing between the sides of a right-angled triangle may be interpreted as follows:

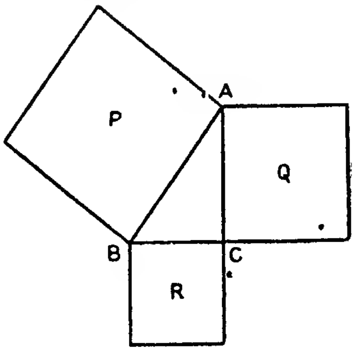


FIG. 34.

Draw a square on each side of the triangle ABC, then area P = area R + area Q (Fig. 34).

$$\therefore\ \text{P sq. ins.} = \text{R sq. ins.} + \text{Q sq. ins.}$$

Referring to equation α, we have

$$\left.\begin{array}{l} c^2 = a^2 + b^2 \\ a^2 + b^2 = c^2. \end{array}\right\}$$ These both have the same mathematical meaning.

or

But if $a^2 + b^2 = c^2$,

then $a^2 = c^2 - b^2, \ldots\ldots\ldots\ldots (\beta)$

$b^2 = c^2 - a^2 \ldots\ldots\ldots\ldots (\gamma)$

Suppose we have a right-angled triangle whose sides are a, b, c. Then, if any *two* sides are known, the third can be determined.

EXAMPLES.

1. Given $a = 3''$, $b = 4''$, to find c.

$$c^2 = a^2 + b^2; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\alpha)$$
$$\therefore \; c^2 = 3^2 + 4^2$$
$$= 9 + 16$$
$$= 25;$$
$$\therefore \; \underline{c = \sqrt{25} = 5''.}$$

2. Given $c = 5''$, $b = 4''$, to find a.

$$a^2 = c^2 - b^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\beta)$$
$$= 5^2 - 4^2$$
$$= 25 - 16$$
$$= 9;$$
$$\therefore \; \underline{a = \sqrt{9} = 3''.}$$

3. Given $c = 5''$, $a = 3''$, to find b.

$$b^2 = c^2 - a^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\gamma)$$
$$= 5^2 - 3^2$$
$$= 25 - 9$$
$$= 16;$$
$$\therefore \; \underline{b = \sqrt{16} = 4''.}$$

The same method of procedure applies if the sides are any engths we choose.

4. Given $a = 39{\cdot}25$ cm., $b = 58{\cdot}38$ cm., to find c.

$$c^2 = a^2 + b^2$$
$$= 39{\cdot}25^2 + 58{\cdot}38^2$$
$$= 1540 + 3409$$
$$= 4949;$$
$$\therefore \; \underline{c = \sqrt{4949} = 70{\cdot}36 \text{ cm.}}$$

5. A ladder 25 ft. long rests against a wall, the top of the ladder being 21 ft. from the ground. What distance is the foot of the ladder from the wall?

$a^2 = c^2 - b^2$; .... (β)

$\therefore\ a^2 = 25^2 - 21^2$
$= 625 - 441$
$= 184$;

Or $25^2 - 21^2$
$= (25-21)(25+21)$
$= 4 \times 46$
$= 184$.

$\therefore\ a = \sqrt{184} = 13{\cdot}56'$.

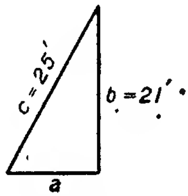


FIG. 35.

6. A guy rope is fixed to a telegraph pole 8′ from the top. The guy is anchored 8·5′ from the foot of the pole. What is its length if the telegraph post is 24′ high?

$c^2 = a^2 + b^2$; .... (a)

$\therefore\ c^2 = 8{\cdot}5 + 16^2$
$= 72{\cdot}25 + 256$;

$\therefore\ c = \sqrt{328{\cdot}25}$
$= 18{\cdot}12'$.

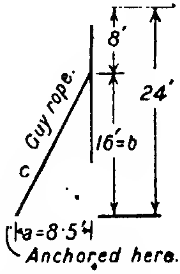


FIG. 36.

7. The back stay of a suspension bridge is 65′ long, and the distance of the anchoring point from the foot of one of the piers 54′. Find the height of the pier.

$b^2 = c^2 - a^2$; .... (γ)

$\therefore\ b^2 = 65^2 - 54^2$
$= 4225 - 2916$
$= 1309$;

Or $65^2 - 54^2$
$= (65-54)(65+54)$
$= 11 \times 119$
$= 1309$.

$\therefore\ b = \sqrt{1309} = 36{\cdot}17'$.

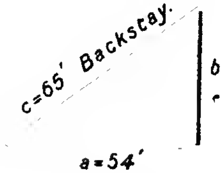


FIG. 37.

8. A rope is fixed to two vertical posts whose bases are 45′ apart. It is 12′ from the base of one and 6′ from the base of

the other. Find the length of the rope (assuming it does not sag) when the posts are on a slope of 1 in 10.

CASE 1.

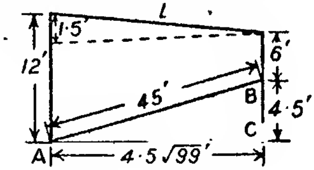


FIG. 38.

AB = 45′, and since the slope (see Appendix) is 1 in 10, BC = $\frac{45}{10}$ = 4·5′.

$$\begin{aligned} AC &= \sqrt{AB^2 - BC^2} \\ &= \sqrt{45^2 - 4{\cdot}5^2} \\ &= 4{\cdot}5\sqrt{10^2 - 1^2} \\ &= \underline{4{\cdot}5\sqrt{99}}. \end{aligned}$$

$$\begin{aligned} l^2 &= 1{\cdot}5^2 + (4{\cdot}5\sqrt{99})^2 \\ &= 2{\cdot}25 + 4{\cdot}5^2 \times 99 \\ &= 2{\cdot}25 + 2004{\cdot}75 \\ &= 2007; \end{aligned}$$

$$\begin{aligned} 4{\cdot}5^2 \times 99 &= 4{\cdot}5^2(100 - 1) \\ &= 4{\cdot}5^2 \times 100 - 4{\cdot}5^2 \times 1 \\ &= 2025 - 20{\cdot}25 \\ &= 2004{\cdot}75. \end{aligned}$$

$$\begin{aligned} \therefore\ l &= \sqrt{2007} \\ &= \underline{44{\cdot}8'}. \end{aligned}$$

CASE 2.

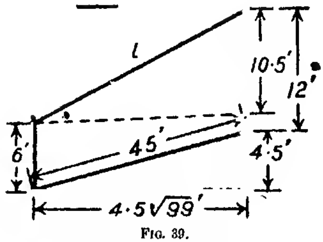


FIG. 39.

$$\begin{aligned} l^2 &= 10{\cdot}5^2 + (4{\cdot}5\sqrt{99})^2 \\ &= 110{\cdot}25 + 2004{\cdot}75 \\ &= 2115; \end{aligned}$$

$$\begin{aligned} \therefore\ l &= \sqrt{2115} \\ &= \underline{46' \text{ about}}. \end{aligned}$$

9. The ratio of the sides about the right angle of a right-angled triangle is 2·35 : 4·15. The hypotenuse is 92·17′ long. Find each side.

$$c^2 = a^2 + b^2, \quad \ldots\ldots\ldots\ldots (1)$$

$$\frac{a}{b} = \frac{2{\cdot}35}{4{\cdot}15}; \quad \ldots\ldots\ldots\ldots (2)$$

$$\therefore \frac{a^2}{b^2} = \left(\frac{2{\cdot}35}{4{\cdot}15}\right)^2;$$

$$\therefore a^2 = \left(\frac{2{\cdot}35}{4{\cdot}15}\right)^2 \times b^2 = 0{\cdot}3207b^2.$$

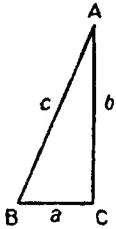


FIG. 40.

Substituting for a^2 in (1), we obtain

$$c^2 = 0{\cdot}3207b^2 + b^2 = 1{\cdot}321b^2 \text{ (practically)};$$

$$\therefore b^2 = \frac{c^2}{1{\cdot}321} = \frac{92{\cdot}17^2}{1{\cdot}321};$$

$$\therefore b = \sqrt{\frac{92{\cdot}17^2}{1{\cdot}321}} = \frac{92{\cdot}17}{\sqrt{1{\cdot}321}}$$

$$= \underline{80{\cdot}26'}.$$

$$a^2 = 0{\cdot}3207b^2;$$

$$\therefore a = b\sqrt{0{\cdot}3207}$$

$$= 80{\cdot}26 \times \sqrt{0{\cdot}3207}$$

$$= \underline{45{\cdot}44'}.$$

Or

$$\frac{a}{b} = \frac{2{\cdot}35}{4{\cdot}15};$$

$$\therefore a = b \times \frac{2{\cdot}35}{4{\cdot}15}$$

$$= \frac{80{\cdot}26 \times 2{\cdot}35}{4{\cdot}15}$$

$$= \underline{45{\cdot}45'}.$$

Examples to be Worked Out.

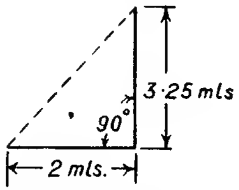
1. (1) $a = 32 \cdot 8$ mm., $b = 89 \cdot 3$ mm. Find c in cms.
(2) $b = 83 \cdot 2$ yds., $c = 193$ yds. Find a in feet.
(3) $a = 32 \cdot 8''$, $b = 2a$. Find c in yds.
Check by drawing each triangle to scale.
(4) An equilateral triangle has a side $3 \cdot 5''$ long. Find the altitude of the triangle.

2. The altitude of a cone is 15″ and the diameter of its base 12″. What is the slant height? Draw a plan and elevation to a scale of $\frac{1}{5}$ full size, and check by measurement.

3. The slant height of a conical tent is $16 \cdot 4'$ and the radius of the base $5 \cdot 8'$. What is the height of the centre pole? Draw views as in (2), and check by measurement.

4. The altitude of a chimney is measured at a point 300 yds. from the base and found to be 285′. How far was the observer from (*a*) the top, (*b*) a point half-way up the chimney. Make a suitable scale drawing.

5. The diagram shows the plan of a road circuiting a common. How many miles would be saved if a man walked across the common as the crow flies? Make a suitable scale drawing. What geometrical truth does this illustrate?



6. In crossing from one side of a river to another, the ferry boat is carried 120 yds. down owing to the current. If the breadth of the river is 280 yds., what is the actual length of the boat's course? Make a suitable scale drawing. (Assume the course to be straight.)

7. A railway incline is 1 in 150. What is the projected or horizontal length? Is the difference between the two worth taking account of in practice?

8. On a certain part of the Snowdon Mountain Railway the gradient is 1 in 6. Through what vertical height will a train ascend, and what horizontal distance will it have traversed, when it has travelled over 185 yds. of rail? Make a scale drawing to check your calculation.

9. The height of a spherical segment is 8·35″ and the radius of the plane end 12·24″. Find the radius of the sphere from which the segment was cut. Draw a plan and elevation (see Figs. 130, 135).

10. A chord 5·35 cms. long is drawn in a circle 13·95 cms. diameter. Calculate how far it is from the centre. Make a drawing to scale (see Figs. 84, 86).

11. The backstay of a suspension bridge is 125 ft. long and the anchoring point is 120 ft. from the base of a pier. Find the height of one of the piers. Make a scale drawing and check.

12. A telegraph pole is 26 ft. high and the guy rope is fixed 7·5 ft. from the top. It is anchored in the ground, 15 ft. from the base of the pole. Find its length. Make a scale drawing and check.

13. The top of a ladder 20 ft. long rests against a wall 16 ft. high. How far is the foot of the ladder from the wall? Make a scale drawing and check.

14. A rope is fixed to two vertical posts whose *bases* are 65 ft. apart. It is 15 ft. from the base of one post and 9 ft. from the base of the other. Find the length of rope (1) when both posts are on horizontal ground, (2) when the posts are on a slope of 1 in 10. Assume there is no sag. (See incline in Appendix.)

15. The Impedance of an electric circuit is given by $I = \sqrt{r^2 + p^2L^2}$. Find I by a graphical construction, when $r = 5$, $p = 100\pi$, $L = 0{\cdot}01$ $(\pi^2 = 10)$.

16. The hypotenuse of a right-angled triangle is 2·35 times the length of one side. Find each side about the right angle, if the hypotenuse is 39·57″ long.

17. The ratio of the sides about the right angle of a right-angled triangle is 1·95 : 3·75. The hypotenuse is 58·56″ long. Find each side.

18. The sum of the sides about the right angle of a right-angled triangle is 18·78″, and the difference 6·53″. Find the three sides of the triangle.

19. A perpendicular is drawn from the vertex to the base of an equilateral triangle 4″ side. Find its length. Check by drawing to scale.

20. If the length of the side in (19) had been $2a$, find the perpendicular, and the ratio of the perpendicular to one side.

21. A perpendicular is drawn from the vertex of an isosceles triangle to the base. Find its length if the sides of the triangle are $3\frac{1}{2}$″, $3\frac{1}{2}$″ and 2″. Check by drawing.

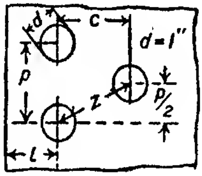
22. Find the square root of 2 geometrically, by drawing a right-angled triangle, sides about the right angle 2″ and 2″. Hence find $\sqrt{3}$.

23. Find $\sqrt{5}$ by a geometrical construction. Hence find $\sqrt{6}$ (see 22).

24. The wheel base of a tram car is 12′ (distance between centres of wheels). Find the horizontal and vertical distances between the wheel centres when the car is on a slope of 1 in 12 (see Appendix regarding slope). Make a suitable scale drawing and check.

25. The diagram represents a plate for a riveted joint, all the holes being the same diameter. Calculate the sizes from the following

formulæ: $p = 2'' + d$, $l = 1\frac{1}{2}d$, $c = 2d$. Draw the given view to scale; measure and calculate z.



SOLUTION OF RIGHT-ANGLED TRIANGLES

SECTION 2.

The right-angled triangle ABC has one right angle C$(=90°)$, and two acute angles A and B, each $< 90°$ (Fig. 41).

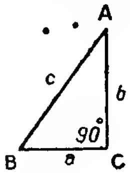


FIG. 41.

Now $$\hat{A} + \hat{B} + \hat{C} = 180;$$

$$\therefore \hat{A} + \hat{B} = 180° - \hat{C}$$
$$= 180° - 90°;$$
$$\therefore \hat{A} + \hat{B} = 90°; \quad \ldots\ldots\ldots\ldots (\alpha)$$
$$\therefore \hat{A} = 90° - \hat{B}; \quad \ldots\ldots\ldots\ldots (\beta)$$
$$\therefore \hat{B} = 90° - \hat{A}. \quad \ldots\ldots\ldots\ldots (\gamma)$$

Thus, if one acute angle be known, the other may be found from (β) or (γ).

EXAMPLES.

1. Given $\hat{A} = 35°$, $\hat{C} = 90°$, to find $\hat{B}$.

$$\hat{B} = 90° - \hat{A}; \quad \ldots\ldots\ldots\ldots (\gamma)$$
$$\therefore \hat{B} = 90° - 35°$$
$$= 55°.$$

2. Given $\hat{B}=82°$, $C=90°$, to find $\hat{A}$.

$$\hat{A}=90°-\hat{B}\,; \quad\dots\dots\dots\dots\dots\dots\dots\dots (\beta)$$
$$\therefore \hat{A}=90°-82.$$
$$=\underline{8°}.$$

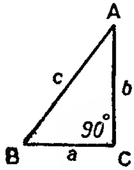


FIG. 42.

Let ABC be any right-angled triangle, $\hat{C}$ being the right angle. In all cases the following definitions are true (Fig. 42):

$$\text{Angle B}\begin{cases}\text{sine} & \hat{B}=b/c=\dfrac{\text{perpendicular}}{\text{hypotenuse}},\\ \text{cosine} & \hat{B}=a/c=\dfrac{\text{base}}{\text{hypotenuse}},\\ \text{tangent} & \hat{B}=b/a=\dfrac{\text{perpendicular}}{\text{base}}.\end{cases}$$

$$\text{Angle A}\begin{cases}\text{sine} & \hat{A}=a/c,\\ \text{cosine} & \hat{A}=b/c,\\ \text{tangent} & \hat{A}=a/b.\end{cases}$$

The above are abbreviated thus:

$$\left.\begin{array}{l}\sin B=b/c,\\ \cos B=a/c,\\ \tan B=b/a.\end{array}\right\}\quad\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$$\left.\begin{array}{l}\sin A=a/c,\\ \cos A=b/c,\\ \tan A=a/b.\end{array}\right\}\quad\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

Notice that $\sin A=\cos B=a/c$

and $\cos A=\sin B=b/c.$

This is always the case when $\hat{A}+\hat{B}=90°$

From (1) we have $\sin B = b/c$. Now, if any two of the three quantities are known, the third may be found thus:

$$\left.\begin{aligned} &\sin B = b/c; \\ &\therefore\ c = b/\sin B; \\ &\therefore\ b = c \sin B. \end{aligned}\right\} \qquad (\delta)$$

Again,
$$\left.\begin{aligned} &\cos B = a/c; \\ &\therefore\ c = a/\cos B; \\ &\therefore\ a = c \cos B. \end{aligned}\right\} \qquad (\epsilon)$$

Also
$$\left.\begin{aligned} &\tan B = b/a; \\ &\therefore\ a = b/\tan B; \\ &\therefore\ b = a \tan B. \end{aligned}\right\} \qquad (\theta)$$

Similar results may be found from (2).

The values of $\sin A$, $\cos A$, $\tan A$, etc., may be obtained from a table of trigonometrical functions.

On reference to the triangle ABC (Fig. 42), we see that it has *three sides* a, b, c, and *three angles* $\hat{A}$, $\hat{B}$, $\hat{C}$. Provided sufficient data are given, the value of each side and each angle can be ascertained, *i.e.* the triangle can be solved.

EXAMPLES.

3. Given $\hat{A} = 25°$, $C = 90°$, $a = 3{\cdot}85$ cms., solve the triangle, *i.e.* find all its sides and angles.

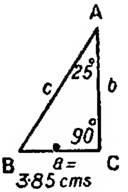


FIG. 43.

$$\begin{aligned} \hat{B} &= 90° - \hat{A} \qquad (\gamma) \\ &= 90° - 25° \\ &= \underline{65°}. \end{aligned}$$

$$b = a \tan \text{B} \; ; \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\theta)$$

$$\therefore \; b = 3{\cdot}85 \tan 65^\circ$$
$$= 3{\cdot}85 \times 2{\cdot}1445$$
$$= \underline{8{\cdot}258 \text{ cms.}}, \text{ say } 8{\cdot}26 \text{ cms. correct to two places.}$$

$$c = \frac{a}{\cos \text{B}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\epsilon)$$
$$= \frac{3{\cdot}85}{\cos 65^\circ}$$
$$= \frac{3{\cdot}85}{0{\cdot}4226}$$
$$= \underline{9{\cdot}112 \text{ cms.}}$$

The complete solution is:

$$\begin{Bmatrix} a = 3{\cdot}85 \text{ cms.,} \\ b = 8{\cdot}26 \text{ cms.,} \\ c = 9{\cdot}11 \text{ cms.,} \\ \hat{\text{A}} = 25^\circ, \\ \hat{\text{B}} = 65^\circ, \\ \hat{\text{C}} = 90^\circ. \end{Bmatrix}$$

4. Given $c = 28{\cdot}95''$, $a = 15{\cdot}25''$, $\text{C} = 90^\circ$, solve the triangle.

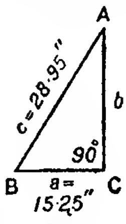


Fig. 44.

$$\cos \text{B} = a/c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$
$$= \frac{15{\cdot}25}{28{\cdot}95}$$
$$= \underline{0{\cdot}5266.}$$

$\therefore$ B is an angle whose cosine is $0{\cdot}5266$.

This may be expressed by writing $\cos^{-1} 0{\cdot}5266 = \hat{B}$;

$$\therefore\ \underline{B = 58^\circ \text{ about.}}$$

(See interpolation in the Appendix if greater accuracy is desired.)

But
$$\hat{A} = 90^\circ - \hat{B}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\beta)$$
$$\therefore\ A = 90^\circ - 58^\circ$$
$$= \underline{32^\circ}.$$
$$b = a \tan B; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\theta)$$
$$\therefore\ b = 15{\cdot}25 \tan 58^\circ$$
$$= 15{\cdot}25 \times 1{\cdot}6$$
$$= \underline{24{\cdot}4''}.$$

The complete solution is:

$$\left\{\begin{aligned} a &= 15{\cdot}25'', \\ b &= 24{\cdot}4'', \\ c &= 28{\cdot}95'', \\ \hat{A} &= 32^\circ, \\ \hat{B} &= 58^\circ, \\ \hat{C} &= 90^\circ. \end{aligned}\right\}$$

5 Given $a = 32{\cdot}19'$, $b = 37{\cdot}56'$, $\hat{C} = 90^\circ$, solve the triangle.

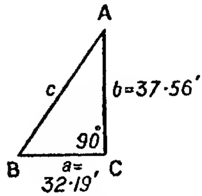


Fig. 45.

$$c^2 = a^2 + b^2; \quad \ldots\ldots\ \ldots\ldots(a) \text{ in Section 1}$$
$$\therefore\ c^2 = 32{\cdot}19^2 + 37{\cdot}56^2$$
$$= 1036 + 1410$$
$$= 2446;$$
$$\therefore\ c = \underline{\sqrt{2446} = 49{\cdot}46'}.$$

$$\frac{b}{a} = \tan B ; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\theta)$$

$$\therefore \quad \tan B = \frac{37 \cdot 56}{32 \cdot 19}$$

$$= 1 \cdot 167 ;$$

$$\therefore \quad B = \tan^{-1} 1 \cdot 167$$

$$= \underline{49 \cdot 5^\circ \text{ about.}}$$

(See interpolation in the Appendix if greater accuracy is desired.)

$$\hat{A} = 90^\circ - \hat{B} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\beta)$$

$$= 90^\circ - 49 \cdot 5^\circ$$

$$= \underline{40 \cdot 5^\circ.}$$

The complete solution is:

$$\left\{\begin{aligned} a &= 32 \cdot 19 \text{ ft.,} \\ b &= 37 \cdot 56 \text{ ,,} \\ c &= 49 \cdot 46 \text{ ,,} \\ \hat{A} &= 40 \cdot 5^\circ, \\ \hat{B} &= 49 \cdot 5^\circ, \\ \hat{C} &= 90^\circ. \end{aligned}\right\}$$

6. The diagram shows the connecting rod and crank of a steam engine in such a position that the thrust in the rod is about a maximum. Find α. If a plan of the arrangement was drawn, determine the projected length of each (Fig. 46).

AB = connecting rod.

BC = crank.

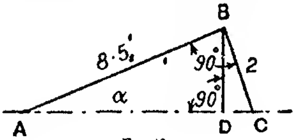


FIG. 46.

$$\frac{2}{8 \cdot 5} = \tan \alpha ;$$

$$\therefore \quad \tan \alpha = 0 \cdot 2353 ;$$

$$\therefore \quad \alpha = \tan^{-1} 0 \cdot 2353$$

$$= \underline{13^\circ \text{ about.}}$$

(See interpolation in Appendix if greater accuracy is desired.)

Projected length of connecting rod = AD.

$$\text{Now}\quad \frac{\mathrm{AD}}{8\cdot5} = \cos 13^\circ;$$

$$\therefore\ \mathrm{AD} = 8\cdot5 \times \cos 13^\circ$$
$$= 8\cdot5 \times 0\cdot9744$$
$$= \underline{8\cdot28'}.$$

$$\hat{\mathrm{A}} + \hat{\mathrm{C}} = 90^\circ;\quad \because\ \mathrm{C} = 90^\circ - \hat{\mathrm{A}}$$
$$= 90^\circ - 13^\circ$$
$$= \underline{77^\circ}.$$

Projected length of crank = DC.

$$\frac{\mathrm{DC}}{\mathrm{BC}} = \cos 77^\circ;$$

$$\therefore\ \mathrm{DC} = \mathrm{BC} \cos 77^\circ$$
$$= 2 \cos 77^\circ$$
$$= 2 \times 0\cdot225$$
$$= \underline{0\cdot45'}.$$

7. The outline of a roof truss with a king post is given. Find the length of the side struts and the height of the king post (Fig. 47).

AB = AC = struts.

AD = king post.

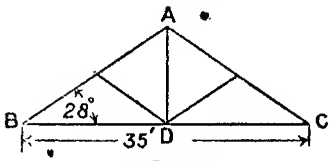


Fig. 47.

$$\frac{\mathrm{AD}}{\mathrm{BD}} = \tan 28^\circ;$$

$$\therefore\ \mathrm{AD} = \mathrm{BD} \tan 28^\circ;$$

$$\therefore\ \mathrm{AD} = 17\cdot5 \times 0\cdot5317,$$

i.e. king post $= \underline{9\cdot31'}$.

$$\frac{\mathsf{BD}}{\mathsf{AB}} = \cos 28^\circ ;$$

$$\therefore \quad \mathsf{AB} = \frac{\mathsf{BD}}{\cos 28^\circ}$$

$$= \frac{17 \cdot 5}{0 \cdot 883},$$

i.e. each strut $= \underline{19 \cdot 82'}$.

8. Find the area of an equilateral triangle, the length of whose side is a.

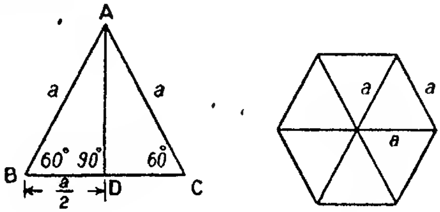


FIG. 48.

Draw AD perpendicular to BC. Then AD bisects BC.

$$\text{Area} = \frac{\mathsf{AD} \times \mathsf{BC}}{2}$$

$$= \mathsf{AD} \times \mathsf{BD}.$$

$$\text{Now} \quad \frac{\mathsf{AD}}{\mathsf{BD}} = \tan 60^\circ ;$$

$$\therefore \quad \mathsf{AD} = \mathsf{BD} \tan 60^\circ$$

$$= \frac{a}{2} \times \sqrt{3} ;$$

$$\text{whence the area} = \frac{a}{2} \times \sqrt{3} \times \frac{a}{2}$$

$$= \underline{\frac{\sqrt{3}}{4} a^2 = 0 \cdot 433 a^2}. \quad (\sqrt{3} = 1 \cdot 732.)$$

Since a regular hexagon* of side a can be divided into six equilateral triangles,

* A plane rectilinear figure with six equal sides.

$$\text{its area} = \frac{6 \times \sqrt{3}}{4} a^2$$

$$= \frac{3\sqrt{3}}{2} a^2$$

$$= \underline{2{\cdot}598a^2}.$$

9. A regular pentagon is inscribed in a circle 10″ diameter. Find the length of each side and the area of the pentagon

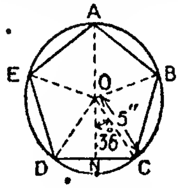


FIG. 49.

Since the sides of the pentagon are equal, the angles subtended at the centre by these sides must also be equal. Hence $D\hat{O}C = E\hat{O}D = E\hat{O}A$, etc.

$$\therefore\ D\hat{O}C = \tfrac{1}{5} \text{ the angle at O}$$

$$= \frac{360}{5}$$

$$= \underline{72^\circ}.$$

If ON is perpendicular to DC, $C\hat{O}N = \tfrac{1}{2} D\hat{O}C$

$$= \frac{72}{2}$$

$$= \underline{36^\circ}.$$

Now, $$\frac{CN}{OC} = \sin 36^\circ;$$

$$\therefore\ CN = OC \sin 36^\circ$$

$$= 5 \times 0{\cdot}5878$$

$$= \underline{2{\cdot}939''}.$$

But $$CN = \tfrac{1}{2} DC;$$

$$\therefore\ DC = 2CN$$

$$= 2 \times 2{\cdot}939,$$

i.e. the length of one side $= \underline{5{\cdot}878''}$.

$$\text{Area of DOC} = \frac{\text{ON} \times \text{DC}}{2}$$

$$= \text{ON} \times \text{CN};$$

$$\frac{\text{ON}}{\text{OC}} = \cos 36^\circ;$$

$$\therefore \quad \text{ON} = \text{OC} \cos 36^\circ$$

$$= 5 \times 0{\cdot}809$$

$$= \underline{4{\cdot}045''}.$$

$$\therefore \quad \text{area of DOC} = 4{\cdot}045 \times 2{\cdot}939$$

$$= \underline{11{\cdot}89 \text{ sq. ins.}}$$

Moreover, there being 5 equal triangles,

$$\text{the area of the pentagon} = 5 \times 11{\cdot}89$$

$$= \underline{59{\cdot}45 \text{ sq. ins}}$$

10. If the pentagon in the last example had been circumscribed, find the length of each side and the area.

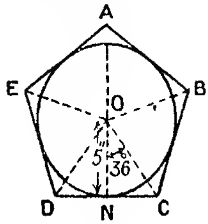


FIG. 50.

As in the last example, $\hat{\text{CON}} = 36^\circ$.

$$\frac{\text{ON}}{\text{ON}} = \tan 36^\circ;$$

$$\therefore \quad \text{CN} = \text{ON} \tan 36^\circ$$

$$= 5 \times 0{\cdot}7265$$

$$= \underline{3{\cdot}6325''}.$$

$$\therefore \quad \text{DC} = 3{\cdot}6325 \times 2,$$

i.e. the length of one side $= \underline{7{\cdot}265''}$.

$$\text{Area of DOC} = \frac{ON \times DC}{2}$$
$$= ON \times CN$$
$$= 5 \times 3{\cdot}6325$$
$$= \underline{18{\cdot}1625 \text{ sq. ins.}}$$

Hence area of pentagon $= 5 \times 18{\cdot}1625$
$= \underline{90{\cdot}8125}$, say $90{\cdot}81$ sq. ins.

11. The diagram ABCD shows the lower portion of a roof truss. If AB = BC = CD, find the length of each, and the angles ABC and BAC (Fig. 51).

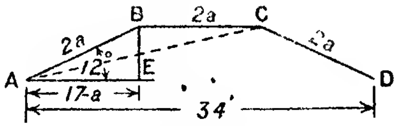


FIG. 51.

$$\frac{17 - a}{2a} = \cos 12^\circ;$$
$$\therefore\ 17 - a = 2a \cos 12^\circ$$
$$= 2a \times 0{\cdot}9781$$
$$= 1{\cdot}956a;$$
$$\therefore\ a + 1{\cdot}956a = 17;$$
$$\therefore\ 2{\cdot}956a = 17;$$
$$\therefore\ a = \frac{17}{2{\cdot}956}$$
$$= \underline{5{\cdot}75'}.$$

Hence $AB = BC = CD = 2a = \underline{11{\cdot}5'}.$

$$A\hat{B}E = 90^\circ - 12^\circ$$
$$= 78^\circ.$$
$$C\hat{B}E = 90^\circ;$$
$$\therefore\ A\hat{B}C = A\hat{B}E + C\hat{B}E = 78^\circ + 90^\circ$$
$$= \underline{168^\circ}.$$

ABC is an isosceles triangle, hence $B\hat{A}C = B\hat{C}A$.

But $$\hat{BAC} + \hat{ABC} + \hat{BCA} = 180°;$$
$$\therefore 2\hat{BAC} + \hat{ABC} = 180°;$$
$$\therefore 2\hat{BAC} = 180° - 168°$$
$$= 12°;$$
$$\therefore \underline{\hat{BAC} = 6°},$$

i.e. AC bisects $\hat{BAE}$.

12. A cylinder 5″ diameter, with its axis vertical, is cut by a plane inclined at 60° to the horizontal. Find the area intercepted by the cutting plane.

The true shape of the section is an ellipse whose major axis is AB and minor axis BC. It is this area which has to be found.

Semi-minor axis = OF = OG
$= 5/2 = 2{\cdot}5''$;
i.e. $\underline{b = 2{\cdot}5}$.

Semi-major axis = OH = OJ = $\frac{AB}{2}$.

Now $\frac{BC}{AB} = \cos 60°$;

$\therefore AB = \frac{BC}{\cos 60°}$
$= \frac{5}{0{\cdot}5}$
$= \underline{10''}$;

$\therefore 2a = 10$
and $\underline{a = 5''}$.

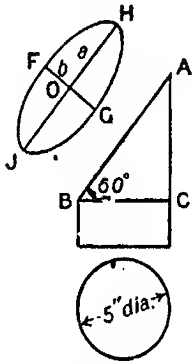


Fig. 52.

Area of ellipse $= \pi ab$ (see p. 90)
$= \pi \times 5 \times 2{\cdot}5$
$= 12{\cdot}5\pi$
$= \underline{39{\cdot}27 \text{ sq. ins.}}$

The area might also have been determined as follows:

$$\frac{\text{Area of section at BC}}{\text{Area of section at AB}} = \cos 60°;$$

$$\therefore \text{ area at AB} = \frac{\text{area at BC}}{\cos 60^\circ}$$

$$= \frac{\pi \times 2{\cdot}5^2}{0{\cdot}5} = \pi \times 5 \times 2{\cdot}5$$

$$= \underline{39{\cdot}27 \text{ sq. ins.,}} \text{ as before.}$$

This principle is applicable whatever the shape of the section. We have $\frac{\text{projected area on H.P.}}{\text{actual area}} = \cos\theta$, where θ is the inclination of the cutting plane to the H.P.

13. The angle of a wedge is 20° and the length of the wedge 5″. What depth must be chosen, so that the volume will be 65 cubic ins.?

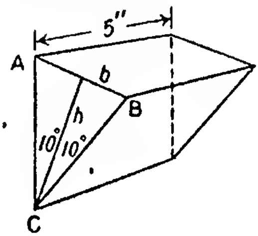


FIG. 53.

Volume = cross-sectional area × length (see page 96)
= area of ABC × 5.

$$\therefore \overset{13}{\cancel{65}} = \left(\frac{b}{2}\right)h \times \cancel{5}.$$

But
$$\frac{b/2}{h} = \tan 10^\circ;$$

$$\therefore b/2 = h \tan 10^\circ;$$

$$\therefore 13 = h \tan 10^\circ \times h;$$

$$\therefore h^2 \tan 10^\circ = 13;$$

$$\therefore\ h^2 = \frac{13}{\tan 10^\circ}$$
$$= \frac{13}{0 \cdot 1763};$$
$$\therefore\ h = \sqrt{\frac{13}{0 \cdot 1763}}$$
$$= \underline{8 \cdot 59''}.$$

14. An indicator rig consists of a rocking lever pivoted at one end, the other end sliding through a guide in the cross-head. The stroke of the engine is 1′ 8″. Find the maximum value of θ, and the length Oa for a diagram 3″ long, *i.e.* $ab = 1\frac{1}{2}''$. What is the reduction? (The indicator cord is parallel to line of stroke.) (Fig. 54.)

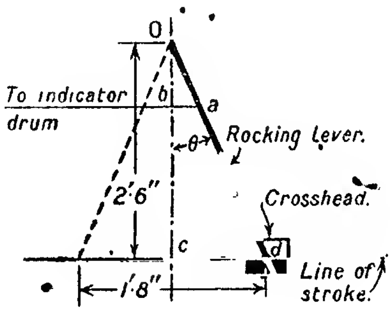


Fig. 54.

$$\frac{cd}{Oc} = \tan\theta;$$
$$\therefore\ \tan\theta = \frac{10}{30} = \frac{1}{3}$$
$$= 0 \cdot 3333 \text{ correct to four places};$$
$$\therefore\ \theta_{max} = \tan^{-1} 0 \cdot 3333$$
$$= \underline{18 \cdot 5^\circ \text{ about.}}$$

(See interpolation in Appendix if greater accuracy is desired.)

$$\frac{ab}{\mathrm{O}a} = \sin\theta\,;$$

$$\therefore\ \mathrm{O}a = \frac{ab}{\sin\theta}$$

$$= \frac{1{\cdot}5}{\sin 18{\cdot}5}$$

$$= \frac{1{\cdot}5}{0{\cdot}3173}$$

$$= \underline{4{\cdot}7''\ \text{about}}$$

for a diagram 3″ long, because the point *a* has a travel of 3″ parallel to *cd*.

$$\text{Reduction} = \frac{\text{length of stroke}}{\text{length of diagram}}$$

$$= \frac{20}{3}$$

$$= \frac{6{\cdot}67}{1}.$$

15. A triangular prism of 3·85″ edge is cut by a plane inclined at 56° to the horizontal. Determine (1) the true area of the section, (2) the area projected on H.P., (3) the true length of $a'b'$ (Fig. 55).

b′r, *bs* and *bd* are not projections of the prism.

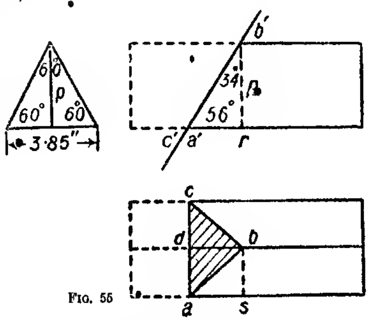


FIG. 55

$$\text{Area of cross-section} = \frac{p \times 3{\cdot}85}{2} = p \times \left(\frac{3{\cdot}85}{2}\right).$$

Now
$$\frac{p}{\frac{1}{2}(3{\cdot}85)} = \tan 60^\circ = 1{\cdot}732\,;$$
$$\therefore\ p = \frac{3{\cdot}85 \times 1{\cdot}732}{2}.$$

Whence, by substitution,
$$\mathrm{A} = \frac{3{\cdot}85 \times \overset{0{\cdot}433}{\cancel{1{\cdot}732}} \times 3{\cdot}85}{\cancel{2} \times \cancel{2}}$$
$$= 3{\cdot}85^2 \times 0{\cdot}433$$
$$= \underline{6{\cdot}419 \text{ sq. ins.}}$$

Let the true area be A_1, then
$$\frac{\mathrm{A}}{\mathrm{A}_1} = \frac{\dfrac{p \times 3{\cdot}85}{2}}{\dfrac{\text{true length of } bd \times ac}{2}}$$
$$= \frac{p \times \cancel{3{\cdot}85}}{a'b' \times \cancel{3{\cdot}85}},$$
$$i.e.\quad \frac{\mathrm{A}}{\mathrm{A}_1} = \frac{p}{a'b'} = \sin 56^\circ\,;$$
$$\therefore\ \mathrm{A}_1 = \frac{\mathrm{A}}{\sin 56^\circ}\,;$$
$$\therefore\ \mathrm{A}_1 = \frac{6{\cdot}419}{0{\cdot}829}$$
$$= \underline{7{\cdot}743 \text{ sq. ins.}}$$

Projected cross-sectional area $= cab$.

Now
$$\frac{cab}{\mathrm{A}_1} = \frac{\dfrac{bd \times \cancel{ac}}{\cancel{2}}}{\dfrac{a'b' \times \cancel{ac}}{\cancel{2}}}$$
$$= \frac{bd}{a'b'}$$
$$= \cos 56^\circ\,;$$
$$\therefore\ cab = \mathrm{A}_1 \times \cos 56^\circ$$
$$= 7{\cdot}743 \times 0{\cdot}5592$$
$$= \underline{4{\cdot}33 \text{ sq. ins.}}$$

True length of $a'b'$ (*i.e.* AB) $= \sqrt{(ab)^2 + (b'r)^2}$
$$= \sqrt{(as)^2 + (bs)^2 + (b'r)^2}.$$

Now $\quad as = a'r,\quad$ and $\quad \dfrac{a'r}{b'r} = \tan 34^\circ.$

But $b'r = p$, $\quad \therefore \dfrac{a'r}{p} = \tan 34°$;

$$\therefore a'r = as = p \tan 34°;$$

$$\therefore as = \frac{3 \cdot 85}{\cancel{2}} \times \cancel{1 \cdot 732}^{\,0 \cdot 866} \times 0 \cdot 6745$$

$$= 3 \cdot 85 \times 0 \cdot 866 \times 0 \cdot 6745$$

$$= \underline{2 \cdot 249''.}$$

$$bs = \frac{ca}{2} = \frac{3 \cdot 85}{2} = 1 \cdot 925''.$$

$$b'r = p = \frac{3 \cdot 85}{\cancel{2}} \times \cancel{1 \cdot 732}^{\,0 \cdot 866};$$

$$\therefore b'r = \underline{3 \cdot 334''.}$$

Whence the true length of $a'b'$,

$$i.e. \quad AB = \sqrt{2 \cdot 249^2 + 1 \cdot 925^2 + 3 \cdot 334^2}$$

$$= \sqrt{5 \cdot 058 + 3 \cdot 705 + 11 \cdot 12}$$

$$= \sqrt{19 \cdot 88\cancel{3}}$$

$$= 4 \cdot 46'' \text{ correct to two places.}$$

The equation
$$\left.\begin{array}{l} \dfrac{A}{A_1} = \sin 56° \\ \quad = \cos 34° \end{array}\right\}$$

might have been written down immediately (see page 42).

Similarly $\dfrac{cab}{A_1} = \cos 56°$ might have been written down without any previous working.

Examples to be Worked Out.

1. Solve the following triangles by drawing and calculation :

(1) $\hat{A} = 32°$,	$\hat{C} = 90°$,	$a = 13 \cdot 21$ yds.
(2) $\hat{A} = 57°$,	$\hat{C} = 90°$,	$a = 21 \cdot 2$ metres.
(3) $c = 82 \cdot 1''$,	$a = 32 \cdot 2''$,	$\hat{C} = 90°$.
(4) $c = 112 \cdot 5$ cms.,	$a = 72 \cdot 4$ cms.,	$\hat{C} = 90°$.
(5) $a = 312 \cdot 6$ mms.,	$b = 252 \cdot 4$ mms.,	$\hat{C} = 90°$.
(6) $a = 121 \cdot 5'$,	$b = 527'$,	$\hat{C} = 90°$.

Draw each triangle to scale.

2. The angle of elevation of a church steeple from a point 150 yds. distant is 12°. Calculate the height of the steeple. Make a scale drawing, and check.

3. The angle of elevation of an aeroplane, 300 yds. distant horizontally, is 82°. Determine its altitude in feet.

4. A regular octagon is inscribed in a circle 12″ diameter. Find its area. Check by a scale drawing.

5. A regular octagon is circumscribed about a circle 12″ diameter. Find its area, check by a scale drawing, and state the ratio of the areas in (4) and (5).

6. A regular n-sided figure, *i.e.* a regular polygon, is inscribed in a circle whose radius is a. Determine its area.

7. If the polygon in (6) had been circumscribed, find its area, also the ratio of the areas in (6) and (7).

8. The volume of a wedge is 73·5 cubic ins., the angle 18° and the depth 10″. What is the length?

9. The angle of a wedge is 16° and the length 4·56″. What depth must be chosen so that the volume will be 56·75 cubic ins.?

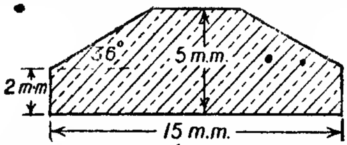
10. A cylinder, 7·35″ diameter, is cut by a plane inclined at 41° to the horizontal. Find the area intercepted by this plane. Draw the cylinder—plan, elevation and true shape of section. Check the length of the major axis from the drawing.

11. The angle of depression of a boat at sea is 18°, the boat being 855 yds. from the shore. Find the altitude of the observer above sea level (in feet).

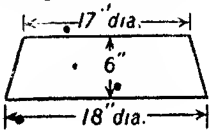
12. The altitude of a right circular cone is 7·65″ and the radius of its base 2·35″. Find its semi-vertical angle.

13. The vertical angle of a conical funnel is 70°. Find the cross-sectional area of the water at the following distances from the vertex: 1, 2, 3, 4, 5 cms. Show that the areas are as $1^2 : 2^2 : 3^2 : 4^2 : 5^2$.

14. The diagram shows the cross-section of a gun-metal oil ring. Find its area in sq. mms. and sq. cms. Make a scale drawing.



15. The diagram shows the elevation of a conical friction clutch. Find the inclination and length of the slant side.



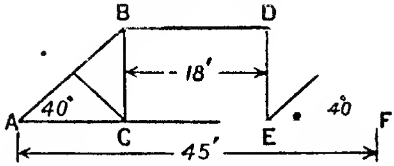
16. The error in a ship's compass is 1°. Assuming the ship to travel in a straight course for 100 nauts (1 naut = 1 nautical mile = 6080′), find how much she has deviated from her true course.

17. A hollow propeller shaft, 15″ external and 8″ internal diameter, is cut by a plane inclined at 51° to the axis of the shaft. Find the area intercepted by the plane. Draw the plan, elevation and true shape of section to scale.

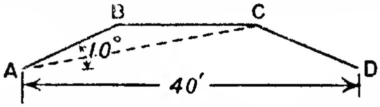
18. The diagram shows a roof truss. Find the lengths of the side struts and the queen posts. Check by a scale drawing.

AB = DF (struts).

BC = DE (queen posts).



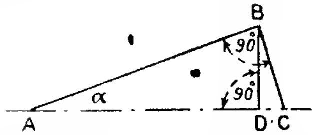
19. The diagram ABCD shows the lower portion of a roof truss. If AB = 0·9BC = CD, find the length of each, and also the angle ABC.



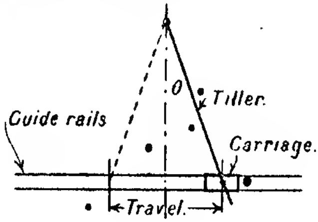
20. The diagram shows the connecting rod and crank in such a position that the thrust in the rod is about a maximum. Find α. If a plan of the arrangement was drawn, determine the projected length of each. Check by a scale drawing.

AB = con. rod = 4·5 cranks long.

BC = crank = 2′6″ long.



21. In the mechanism known as Rapson's Slide for the steering gear of ships, the arrangement is as shown.



Find the travel of the carriage for an angular variation of 70°, *i.e.* $\theta = 35°$.

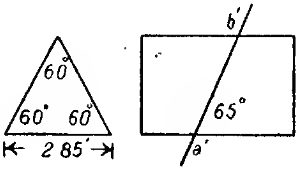
22. A line, whose plan is 7·32″ long, is inclined at 37° to the horizontal plane. Find its true length.

23. A line, whose elevation is 19·85″ long, is inclined at 53° to the vertical plane. Find its true length.

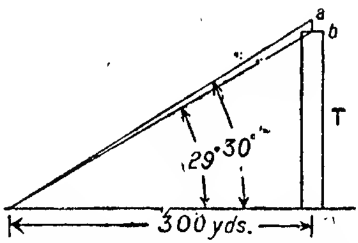
24. A line is 85·97 mm. long, and is inclined at 41° to the H.P. Calculate the length of its plan.

25. The area of a certain district, as shewn on a map, is 85·92 sq. miles, and the average slope, *i.e.* inclination, is 7°. What is its true area?

26. A triangular prism is cut by a plane inclined at 65° to H.P. Determine the true area of the cross section if the side of the base is 2·85″. Find also the true length of $a'b'$. Draw a plan of the section and find its area.

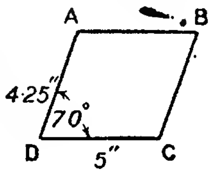


27. ab is a flagstaff on the top of a tower T. Find ab in feet.



28. Find the radii of the circles inscribed in and circumscribed about an equilateral triangle of side $2a$. Find also the ratio of the areas of the circles.

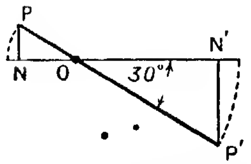
29. A piece of metal has to be cut to the following shape from a rectangular plate. Find the least area of the original plate. What is the area of ABCD, and also the ratio of the two areas?



30. A crane can turn horizontally through 120°. Determine what projected area it can work if the maximum and minimum inclinations of its jib are 70° and 40°. The jib is 30′ long.
(The projected area is a portion of a ring. See page 76).

31. The plan of a crane jib is 20′ long when the inclination of the jib is 50°. Find the least length of chain necessary in this position, assuming it to be along the jib, pass over the pulley at the end and touch the ground. Allow for it being coiled 4 times round a drum 12″ diameter.

32. A rocking lever has arms 6″ and 15″. How far does each end move vertically when the lever turns through 30° from the horizontal, *i.e.* find PN and P′N′.

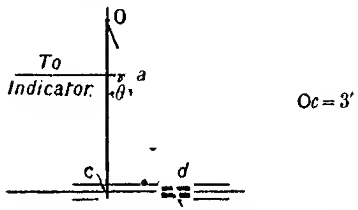


33. A ladder is 30′ long, and its maximum and minimum inclinations to the horizon are 75° and 50°. Find the maximum and minimum heights of wall that can be scaled, and the distance of the foot of the ladder from the wall in each case. Check by scale drawings.

34. The sum of the two sides about the right angle of a right-angled triangle is 84′, and the difference 12′. Solve the triangle.

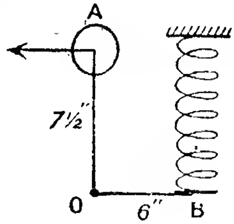
35. A force of 150 lbs. acts at 40° to the horizontal. Find its horizontal and vertical components, graphically and analytically, *i.e.* by drawing and calculation.

36. An indicator rig consists of a rocking lever pivoted at one end, the other end sliding through a guide at the crosshead. The stroke of the engine is 2′, *i.e.* $cd = 1'$. Find the maximum value of θ, and the length of Oa if the diagram is $2\frac{1}{2}''$ long. What is the reduction? Check by a scale drawing.



37. A right elliptical cylinder, major axis 8·2″, minor axis 6·4″, is cut by a plane at 49° to H.P. Determine the area intercepted by the plane.

38. The sketch represents the Wilson Hartnell governor diagrammatically. Find the radius of the ball path if the spring extends 1″. Check by a scale drawing. (The lever AOB turns about O, and B is depressed 1″ vertically.)



CHAPTER III.

THE CIRCLE.

SECTION 1.

A CIRCLE is a plane figure such that every part of its boundary line is the same distance from a fixed point called the centre. The boundary line of the figure is called the circumference, and all points on it are equidistant from the centre O. A line drawn from O to any point on the circumference (such as OP) is a radius, and one drawn through the centre and terminated at each extremity by the circumference, is a diameter, *e.g.* NOM (Fig. 56).

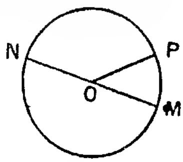


FIG. 56.

The circle is the most easily described curvilinear figure. Since it is a closed curve it envelops an area.

The length of its circumference is known as the perimeter.

In all problems involving the circle or portions of it, a quantity denoted by the Greek letter π is found. Its value cannot be estimated exactly. Approximations to the value of π are (1) $\frac{22}{7}$, (2) $3{\cdot}142$, (3) $3{\cdot}1416$, and any of these may be used for the purposes of calculations performed by engineers.

Perimeter of a circle $= 2\pi r$, where r is the radius. But $2r = d$, the diameter; $\therefore$ perimeter $= \pi d$.

Hence, the constant π is the ratio of the perimeter of any circle to its diameter,* both being measured in the same units.

* Since $P = \pi d$, it follows that $\pi = \frac{P}{d}$.

$$\text{Area} = \pi r^2, \text{ but } r = \frac{d}{2};$$

$$\therefore\ A = \pi\left(\frac{d}{2}\right)^2$$

$$= \frac{\pi d^2}{4}$$

$$= \underline{0{\cdot}7854 d^2}, \text{ since } \frac{\pi}{4} = \frac{3{\cdot}1416}{4} = 0{\cdot}7854.$$

Per. $= \pi d$ and $A = \frac{\pi d^2}{4}$ or $0{\cdot}7854 d^2$ are the formulae generally adopted by engineers. It is more convenient to take $A = 0{\cdot}7854 d^2$ than to take $A = \frac{\pi d^2}{4}$ for purposes of calculation, but it is well to know that $0{\cdot}7854 = \frac{\pi}{4}$, since π is an indispensable factor in calculations on the circle.

Ring or Annulus. The diagram (Fig. 57) shews two *concentric* (same centre) circles. If A_1 = area of outer and A_2 = area of inner, the area enclosed between the two is

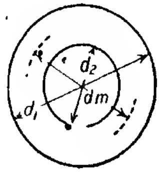


FIG. 57.

$$A_1 - A_2 = \pi r_1^2 - \pi r_2^2$$
$$= \pi(r_1^2 - r_2^2) = \pi(r_1 - r_2)(r_1 + r_2)$$
$$= \frac{\pi}{4}(d_1^2 - d_2^2) = \frac{\pi}{4}(d_1 - d_2)(d_1 + d_2)$$
$$= \underline{0{\cdot}7854(d_1^2 - d_2^2)}$$
$$= \underline{0{\cdot}7854(d_1 - d_2)(d_1 + d_2)}.$$

The area may also be expressed thus:

$$A_1 - A_2 = \frac{\pi}{4}(d_1^2 - d_2^2)$$

$$= \pi\left(\frac{d_1 + d_2}{2}\right)\left(\frac{d_1 - d_2}{2}\right)$$

$= \underline{\pi d_m t}$, d_m being the mean diameter and t the thickness of the ring,

$=$ $\underline{\text{mean circumference} \times \text{thickness}}$.

If the circles are *eccentric i.e.* different centres, one of them being wholly inside the other, the area is given by

$$\frac{\pi}{4}(d_1^2 - d_2^2) \text{ or } 0{\cdot}7854(d_1^2 - d_2^2).$$

The distance between their centres, *i.e.* $0_1 0_2$, is termed the *eccentricity* (Fig. 58).

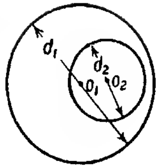


Fig. 58.

EXAMPLES.

1. A circle is 12·3 ins. dia. What is (1) its perimeter, (2) its area?

$$\begin{aligned}(1)\ \mathrm{P} &= \pi d \\ &= \pi \times 12{\cdot}3 \\ &= \underline{38{\cdot}65''.}\end{aligned}$$

$$\begin{aligned}(2)\ \mathrm{A} &= \frac{\pi d^2}{4} \\ &= \frac{\pi \times 12{\cdot}3^2}{4} \\ &= \underline{119{\cdot}1 \text{ sq. ins}}\end{aligned}$$

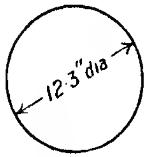


Fig. 59.

2. A cyclist travels at the rate of 10 miles per hour. The bicycle wheels are each 28 ins. dia. Find how many times each revolves (1) per min., (2) per mile, assuming that no slip occurs.

$$60 \text{ m.p.h.} = 88 \text{ ft. per sec.};$$

$$\therefore 10 \quad " \quad = \frac{88}{6} \quad "$$

$$\therefore 10 \quad " \quad = \frac{88 \times \cancel{60}^{10}}{\cancel{6}} \text{ ft. per min.}$$

$$= \underline{880 \text{ ft. per min.}}$$

$$\text{Per. of wheel} = \pi d = \frac{\cancel{28}^{7}\pi}{\cancel{12}_{3}} = \frac{\cancel{7} \times 22}{\cancel{7} \times 3} = \frac{22}{3} \text{ feet.}$$

$$\text{Revs. per min.} = \frac{\text{distance travelled per min.}}{\text{per. of wheel}}$$

$$= \frac{\overset{40}{\cancel{880}}}{\frac{\cancel{22}}{3}}$$

$$= 40 \times 3$$

$$= \underline{120}.$$

Notice how convenient it is to take $\pi = \frac{22}{7}$ in this problem.

$$\text{Revs. per mile} = \frac{5280 \text{ ft.}}{\text{per. of wheel}}$$

$$= \frac{\overset{240}{\cancel{5280}}}{\frac{\cancel{22}}{3}}$$

$$= 240 \times 3$$

$$= \underline{720}$$

The last result might also have been obtained as follows:

Time taken per mile at 10 m.p.h. = 6 mins.

But r.p.m. = 120;

$\therefore$ revs. in 6 min. $= 6 \times 120$

$= \underline{720}.$

3. A flywheel 10·5′ dia. makes 150 r.p.m.; find the peripheral speed* in ft. per min., ft. per sec., and miles per hour.

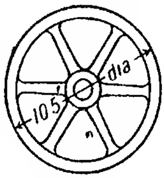


Fig. 60.

R.p.m. = 150.

Distance passed over by a point on the rim in one rev.

$= \pi \times 10{\cdot}5$ ft.

* The speed of a point on the circumference.

$$\text{Distance in one min.} = \text{r.p.m.} \times (\pi \times 10{\cdot}5)$$
$$= 150\pi \times 10{\cdot}5$$
$$= 1575\pi,$$

i.e. peripheral speed $= \underline{4949 \text{ ft. per min.}}$, 4950 ft. per min. say,

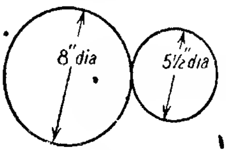
$$= \frac{4949}{60}$$
$$= \underline{82{\cdot}5 \text{ ft. per sec. about.}}$$

Now, 88 ft. per sec. = 60 m.p.h. ;

$$\therefore\ 82{\cdot}5 \text{ ft. per sec.} = \frac{82{\cdot}5 \times 60}{88} = \frac{247{\cdot}5}{4{\cdot}4} = 56{\cdot}25,$$

$\therefore$ peripheral speed $= \underline{56{\cdot}25 \text{ miles per hour about.}}$

4. Two toothed wheels having pitch circles 8″ and $5\frac{1}{2}$″ gear together. If the centres are increased $1\frac{1}{4}$ ins., find the diameters of the new pitch circles to obtain the same angular velocity ratio.



d_1 and d_2 are diameters.
n_1 and n_2 are r.p.sec.

FIG. 61.

Peripheral speed of (1) = peripheral speed of (2).

$$\therefore\ \pi d_1 n_1 = \pi d_2 n_2 ;$$
$$\therefore\ 2\pi r_1 n_1 = 2\pi r_2 n_2 ;$$
$$\therefore\ \frac{2\pi n_1}{2\pi n_2} = \frac{r_2}{r_1} = \text{angular velocity ratio (see page 68)};$$
$$\therefore\ \text{velocity ratio} = \frac{d_2}{d_1}\left(= \frac{r_2}{r_1}\right)$$
$$\text{and}\quad \frac{d_2}{d_1} = \frac{8}{5\frac{1}{2}};$$
$$\therefore\ \frac{r_2}{r_1} = \frac{16}{11}.$$

After the centres have been altered,

$$r_1 + r_2 = 4 + 2 \cdot 75 + 1 \cdot 25 ;$$

$$\therefore\ r_1 + r_2 = 8''. \quad \ldots\ldots\ldots\ldots (\alpha)$$

$$\text{Now}\ \frac{r_2}{r_1} = \frac{16}{11} ;$$

$$\therefore\ 11 r_2 = 16 r_1,$$

$$\left.\begin{array}{ll}\text{or} & 11 r_2 - 16 r_1 = 0, \\ \text{or} & 16 r_1 - 11 r_2 = 0.\end{array}\right\} \quad \ldots\ldots\ldots\ldots (\beta)$$

To find r_1 and r_2, equations (α) and (β) must be solved.*

$$r_1 + r_2 = 8 ; \quad \therefore\ 16 r_1 + 16 r_2 = 128$$

$$16 r_1 - 11 r_2 = \ \ 0 \ldots\ldots\ldots\ldots (\beta)$$

Subtracting,

$$27 r_2 = 128$$

$$\therefore\ r_2 = \tfrac{128}{27}$$

$$= \underline{4 \cdot 741''}.$$

$$\therefore\ d_2 = 2 \times 4 \cdot 741$$

$$= \underline{9 \cdot 482''}.$$

$$r_1 + r_2 = 8 ;$$

$$\therefore\ r_1 = 8 - r_2$$

$$= 8 - 4 \cdot 741$$

$$= 3 \cdot 259'' ;$$

$$\therefore\ d_1 = 2 \times 3 \cdot 259$$

$$= \underline{6 \cdot 518''}.$$

The diameters just found, viz. 9·482″ and 6·518″, would give the necessary angular velocity ratio. Since the circumferences of the pitch circles will now be longer, the number of teeth in each wheel and also the pitch will have to be reconsidered and a suitable selection made. Notice that the diameters are given to three decimal places. This is for machine-cut teeth.

5. The pitch circle of a toothed wheel is 27 ins. dia. and the number of teeth 85. Find the pitch of the teeth to the third decimal place.

* α and β are simultaneous equations. For methods of solution see any elementary algebra book.

If A and B are the centres of two consecutive teeth, the arc AB is the pitch.

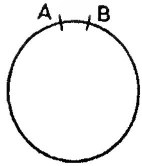


FIG. 62.

$$\text{Pitch} = \frac{\text{perimeter of pitch circle}}{\text{number of teeth}} = \frac{\pi d}{n}$$

$$= \frac{\pi \times 27}{85}, \qquad \text{NOTE.}—d = \frac{pn}{\pi}$$

$$i.e.\ p = \underline{0 \cdot 998''}. \qquad = \underline{0 \cdot 3183 pn.}$$

Notice that if p were made $1''$, the pitch of the 85^{th} tooth would be $84 \times 0 \cdot 002 = 0 \cdot 168''$ too small. In practice the pitch would be made $1''$ and the dia. of the wheel increased slightly. The new dia. would be $\frac{85 \times 1}{\pi} = 27 \cdot 057''$. If teeth are moulded, diametral dimensions given to the second decimal place are adequate.

6. The diameter of the high pressure cylinder of a marine engine is $24''$, and the effective steam pressure at a certain instant is 45 lbs. per sq. in. Find the force urging the piston, etc., downwards—due to steam pressure.

Force due to steam pressure

= area of cyl^r in sq. in. × effective press. in lbs. per sq. in.

$$= \frac{\pi d^2}{4} \times p$$

$$= \frac{\pi \times 24 \times \overset{6}{\cancel{24}} \times 45}{\cancel{4}}$$

$$= \pi \times 144 \times 45$$

$$= \underline{20{,}360 \text{ lbs., say } 20{,}400 \text{ lbs.}}$$

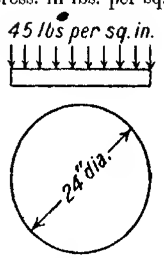


FIG. 63.

Notice that the product of sq. in. units and lbs. per sq. in. units gives lb. units, thus:

$$(\cancel{\text{ins.}}) \times (\cancel{\text{ins.}}) \times \frac{\text{lbs.}}{(\cancel{\text{ins.}}) \times (\cancel{\text{ins.}})} = \text{lbs.}$$

7. A bolt has to be made so that the area of the unscrewed part is equal to the area of the screwed part at the bottom of the thread, by drilling a hole axially through the centre of the unscrewed part.* Find the dia. of the hole for a $\frac{3}{4}''$ bolt, the dia. at the bottom of the thread being 0·622″.

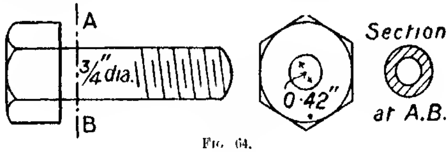


Fig. 64.

Let d_1 = external dia. of unscrewed part = dia. at top of thread.
,, d_2 = internal ,, ,, ,, = dia. of hole.
,, d_3 = dia. at bottom of thread.

Then if area of unscrewed part = area at bottom of thread, we have

$$\frac{\pi}{4}d_1^2 - \frac{\pi}{4}d_2^2 = \frac{\pi}{4}d_3^2;$$

$$\therefore \frac{\pi}{4}(d_1^2 - d_2^2) = \frac{\pi}{4}d_3^2;$$

$$\therefore d_1^2 - d_2^2 = d_3^2;$$

$$\therefore d_2^2 = d_1^2 - d_3^2;$$

$$\therefore d_2 = \sqrt{d_1^2 - d_3^2}$$

$$= \sqrt{0{\cdot}75^2 - 0{\cdot}622^2}$$

$$= \sqrt{0{\cdot}5625 - 0{\cdot}387}$$

$$= \sqrt{0{\cdot}1755}$$

$$= \underline{0{\cdot}419''.}$$

The problem may be solved by a geometrical construction as shown below:

AB = d_1 = dia. at top of thread.
BC = d_2 = ,, of hole.
AC = d_3 = ,, at bottom of thread.

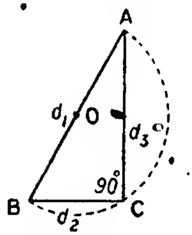


Fig. 65.

* Such bolts are termed bolts of uniform strength.

ABC is a right angled triangle constructed as follows: Set off AB $= d_1$ to scale: bisect AB at O and draw the semicircle ABC. Cut off the chord AC $= d_3$ to scale. Join BC. Then ABC is a right-angled triangle, the angle $\hat{C}$ being 90° * and BC $= d_2$ to scale.

We have $d_1^2 = d_2^2 + d_3^2$ (from the figure);

$$\therefore\ d_2^2 = d_1^2 - d_3^2;$$

$$\therefore\ d_2 = \sqrt{d_1^2 - d_3^2}$$

$$= \sqrt{\text{AB}^2 - \text{AC}^2}$$

$$= \underline{\text{BC}}.$$

8. A boiler end plate is 7′ 6″ dia., and has one hole for a furnace tube 3′ 3″ dia. If the thickness of the end plate is $\frac{3}{4}$″, find its weight. 1 sq. ft. of $\frac{1}{8}$″ steel plate = 5·1 lbs.

Net area of end plate

$$= \frac{\pi}{4}(d_1^2 - d_2^2)$$

$$= \frac{\pi}{4}(7{\cdot}5^2 - 3{\cdot}25^2)$$

$$= \frac{\pi}{4}(56{\cdot}25 - 10{\cdot}56)$$

$$= \frac{\pi}{\cancel{4}} \times \overset{11\ 42}{\cancel{45{\cdot}69}}$$

$$= \underline{35{\cdot}88 \text{ sq. ft.}}$$

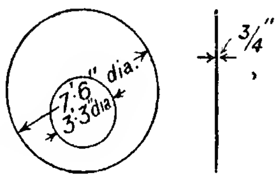


FIG. 66.

Weight of end plate = net area × wt. of 1 sq. ft. of $\frac{3}{4}$″ plate

$$= 35{\cdot}88 \times 5{\cdot}1 \times 6$$

$$= 35{\cdot}88 \times 30{\cdot}6$$

$$= \underline{1098 \text{ lbs.—about } 1100 \text{ lbs.}}$$

Presuming that the above dimensions are correct, we are quite justified in saying that the weight is about 1100 lbs., for the plate might neither be uniform in thickness nor homogeneous † in structure. No very serious error would be committed if the weight was taken as $\frac{1}{2}$ ton.

* The angle in any semicircle is 90°.

† The same throughout.

9. The "lead" of a (single) square screw thread is 1″ and the mean diameter 4″. Find the mean length of 8 convolutions and the angle of the thread.

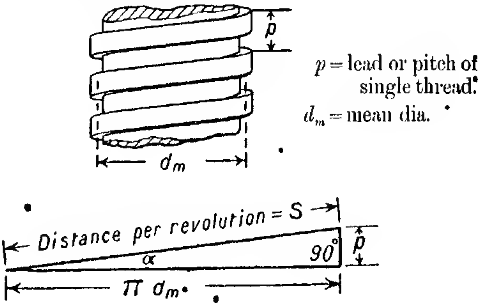


FIG. 67.

The "lead" is the distance the nut advances along the screw when it has made one revolution.

Now a point on the nut has two motions. (1) It moves round the screw in circular motion, (2) it moves along the screw in straight-line motion.

The distance it travels in circular motion is πd_m, and the distance in straight-line motion is p.

But the directions of these two motions are at 90° to one another; hence the distances may be represented by a right-angled triangle.

We have $$s^2 = (\pi d_m)^2 + p^2;$$

$$\therefore\ s = \sqrt{(\pi d_m)^2 + p^2}$$
$$= \sqrt{(\pi \times 4)^2 + 1^2}$$
$$= \sqrt{158 + 1}$$
$$= \sqrt{159},$$

i.e. the length of 1 convolution $= \underline{12{\cdot}61''}$.

The length of 8 convolutions $= 8 \times 12{\cdot}61$
$$= \underline{100{\cdot}88'' \text{—about } 101''}.$$

Observe that $$\pi d_m = \pi \times 4 = 3{\cdot}1416 \times 4$$
$$= \underline{12{\cdot}5664, \text{ say } 12{\cdot}57},$$

which is not far short of 12·61. Evidently, the distance travelled in circular motion is practically the same as that travelled in helical or screw motion.

$$\tan \alpha = \frac{p}{\pi d_m}$$

$$= \frac{1}{12 \cdot 57}$$

$$= 0 \cdot 0796;$$

$$\therefore \alpha = \tan^{-1} 0 \cdot 0796$$

$$= \underline{4^\circ 33'}. \quad \text{(See interpolation in App.)}$$

Or $\tan \alpha = \frac{1}{12 \cdot 57}$.

But $\alpha = \tan \alpha$ when α is small (see Appendix).

Hence $\alpha = \frac{1}{12 \cdot 57}$ radian

$$= \frac{57 \cdot 3}{12 \cdot 57}$$

$$= 4 \cdot 559^\circ$$

$$= \underline{4^\circ 34' \text{ about}},$$

which agrees very closely with the above result.

Examples to be Worked Out.

1. Find the perimeter and area of each of the following circles:

(*a*) rad. = 5·188 cm.	(*d*) dia. = 9·25″.
(*b*) ,, = 48·19″.	(*e*) ,, = 25·25′.
(*c*) ,, = 42·13 yds.	(*f*) ,, = 100 metres.

2. The wire from a signal cabin to a signal is 450 yds. long, and the guide pulleys on the posts are $1\frac{1}{2}''$ diameter. Assuming that the wire is pulled 12″ to cause the signal to drop, determine how many revolutions each pulley makes, no allowance being made for slip or stretch of wire.

3. What is the cross-sectional area of an engine cylinder 19·5″ diameter? If the pressure of steam at a certain instant is 195 lbs. per sq. in., what force is tending to blow the cylinder cover off?

4. The diameter of a lever safety valve is 3″, and the steam blows off at 95 lbs. per sq. in. Determine the upward pressure on the valve. (Area × pressure per sq. in. = total pressure.)

5. A deadweight safety valve is loaded with 225 lbs., and the steam blows off at 85 lbs. per sq. in. Find the diameter necessary at the valve seat. (Area × pressure = load.)

6. The gudgeon pin of a gas engine is $3\frac{1}{4}$″ diameter. What is the area resisting shear, *i.e.* twice the cross-sectional area?

7. A cyclist travels at 14 miles per hour. Find how many revolutions his bicycle wheel makes (1) per mile, (2) per minute, if its diameter is 28 ins.

8. A flywheel 15′ diameter revolves 75 times per minute. Find the peripheral speed in feet per minute, feet per second and miles per hour.

9. A Lancashire boiler is 8′ diameter, and the front end plate is pierced by two holes 3′ 3″ diameter for furnace tubes. Find the area and weight of metal if the thickness is $\frac{7}{8}$″. 1 sq. ft. of $\frac{1}{8}$″ steel plate = 5·1 lbs.

10. A shaft for driving machinery in a factory is $3\frac{3}{4}$″ diameter. What is the cross-sectional area?

11. A hollow propeller shaft, 17 ins. external and $7\frac{1}{2}$ ins. internal diameter, is used on board a certain vessel. What is the cross-sectional area?

12. The diameter of a cylindrical winch barrel for a crane is $9\frac{1}{2}$″. How many coils and what length of barrel would be necessary if the maximum height through which loads were lifted was 22′? Rope = 1″ diameter.

13. If 3′ was added to the diameter of the earth, by how much would its circumference increase?

14. The pitch circle of a toothed wheel is $27\frac{3}{8}$″ diameter, and the pitch of the teeth 1″. Find the number of teeth.

15. The pitch circle of a toothed wheel is 3′ 3″ diameter, and the number of teeth 65. Find the pitch of the teeth.

16. The perimeter of a circle is 63·22 ins. and its area 318·1 sq. ins; find its diameter without extracting a square root or dividing by π.

17. The diameter of the contrivance known as the Joy Wheel is 27′. If the wheel revolves 2·75 times before those at the circumference are shot off, find how far they are carried round.

18. A length of wire has to be selected for railings round part of a bowling green. The style of railings and the dimensions are given. Calculate the length of wire per set.

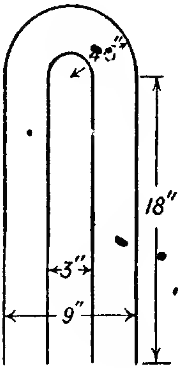


Fig. for Ex. 18.

19. In the oiling arrangement of an electric motor, a ring 7″ diameter revolves with a shaft 4″ diameter and dips into an oil well. Assuming no slip, find the revolutions per minute of the ring when the shaft makes 650. What would be the revolutions of ring for 80% slip?

20. Two toothed wheels connecting two spindles 8·325″ apart must have an angular velocity ratio of 3 : 1. Find the size of each pitch circle to the 3rd decimal place.

21. Two toothed wheels having pitch circles 35·7″ and 10·89″ diameters gear together. Owing to alteration of machinery the centres have to be increased 1¼″. Find the necessary diameters to preserve the same angular velocity ratio.

22. A closely coiled helical spring has 18 turns, each 2¼″ mean diameter. Find its length approximately.

23. The front end plate of a Lancashire boiler is 8′ diameter, and is pierced by two holes 3′ 3″ diameter for furnace tubes. Calculate the force tending to blow the front end plate off when the pressure is 130 lbs. per sq. in. (Force = net area of end plate × pressure.)

24 A lifting gear consists of a drum and a handle 2′ 6″ diameter fixed to the drum. When the handle is turned, a rope is wound on the drum. When the extremity of the handle has moved 60′, 9′ of rope has been wound on. Find the diameter of the drum.

25. A thrust bearing for a propeller shaft has 12 collars, each 8″ maximum and 5½″ minimum diameter. The pressure per sq. in. on one side of each collar is 80 lbs. Calculate the thrust on the propeller. (Find the area of a hollow circle, 8″ external diameter and 5½″ internal diameter, and multiply by the pressure per sq. in.)

26. A bolt has to be made so that the area of the unscrewed part is equal to the area of the screwed part at the bottom of the thread, by drilling a hole axially through the unscrewed part. Find the diameter of this hole for a 1½″ bolt, the diameter at the bottom of the thread being 1·287″.

27. The earth rotates on its axis once in 24 hours. Find the velocity of a point on the equator in miles per minute. The diameter at the equator is 7920 miles.

28. A hollow rod, of uniform thickness throughout its length, has a taper of 1 in 20. It is 2″ internal diameter and 3″ external diameter at the small end. Find the cross-sectional area at the large end if the rod is 20″ long. Draw a plan and elevation to scale. (See Appendix regarding taper.)

29. The peripheral speed (the speed of a point on the circumference) of a flywheel must not exceed 4800 ft. per min. What is the maximum diameter that a flywheel can have which makes 110 r.p.m.?

30. A motor pulley 8″ diameter drives a line of shafting by means of a leather belt. The pulley on the shafting is 24″ diameter. If the motor pulley makes 710 r.p.m., find the r.p.m. of the shafting on the assumption that the belt does not slip.

31. If in (30) the shafting makes 180 r.p.m., what size of pulley would be requisite?

32. Four toothed wheels, having 58, 24, 96 and 74 teeth respectively, gear together. The pitch of the teeth in each wheel is $\frac{1}{2}''$. Determine the diameter of each pitch circle.

33. A solid shaft 9″ diameter has the same strength as a hollow shaft 10″ external diameter and 7·22″ internal diameter. Compare the cross-sectional areas of the two shafts. (This is also a comparison of their weights, provided their lengths are the same.)

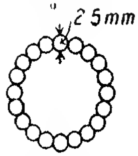
34. A locomotive driving wheel is 6′ 6″ diameter, and makes 100 r.p.m. Find the actual speed of the train in miles per hour when the slip of the wheel is 6·5 %. How is slip prevented?

35. The "lead" of a single square screw thread is 1″ and the mean diameter $3\frac{1}{2}''$. Find the mean length of 5 convolutions.

36. A circle is described about a square $5\dot{5}''$ side. Find its radius and the ratio of the areas.

37. A square is inscribed in a circle 7·5″ diameter. Find the side of the square and the ratio of the areas.

38. Part of the section of an electric cable consists of 20 circles, each 2·5 mms. diameter, arranged thus:



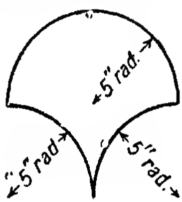
Find the diameters of the inscribed and circumscribed circles.

39. The cross-sectional area of a steam engine cylinder is 320 sq. ins. Find its diameter to the nearest $\frac{1}{4}''$.

40. A high-speed engine makes 450 revs. per min. The length of the crank is 6″. Find the mean piston speed in ft. per min. (The mean piston speed is the distance the piston travels backwards and forwards in one minute.)

41. A locomotive driving wheel is 6′ 6″ diameter, and makes about 259 revs. per min. when travelling at 60 miles per hour, there being no slip. If the stroke is 26″, find the mean piston speed in feet per min., *i.e.* the distance it travels backwards and forwards in one minute.

42. Find the area of the given pelecoid, and deduce a formula giving the area in terms of the radius r.



43. A piece of sheet iron is 30′ wide before corrugation. Find the approximate number of corrugations and the width after corrugation. The corrugations are $1\frac{1}{2}''$ radius and are semi-circular.

44. The cross-sectional area of a hollow cast-iron column is 201 square inches, and the ratio of the external to the internal diameter 5 : 3. Find each diameter.

45. Two circles, 8″ diameter and 4″ diameter, have an eccentricity of $1\frac{1}{2}''$. Draw them to scale, and calculate the area included between them.

46. The perimeter of a semi-circular segment is 25″, *i.e.* curved portion + straight portion. Find the radius of the segment.

47. A square has the same perimeter as a circle 10″ diameter. Shew that the ratio of the area of the square to the area of the circle is $\frac{\pi}{4} = 0{\cdot}7854$.

48. The area of a circle is 8 times its perimeter. Find the diameter. (The dimensions are inches.)

49. A steel bar of square section is equal in area to a rod 5″ diameter. Find the side of the square.

50. In example (49), if the side of the square had been 5″, what would have been the diameter of the circle?

SECTION 2.

Circular Arc. The length of a circular arc is given by: arc $= r\theta$, r being the radius and θ the angle subtended at the centre in radians (Fig. 68).

Now 2π radians $= 360$ degrees;

$$\therefore \ 1 \text{ radian} = \frac{360}{2\pi} \text{ degrees,}$$

i.e. $\underline{1^c = 57{\cdot}3^\circ \text{ approximately.}}$

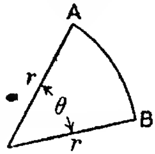


FIG. 68.

2π is the radian measure of a circle, and in the above formula, if $\theta = 2\pi$, the arc $= 2\pi r$; *i.e.* the perimeter of a circle.

Angle in degrees	30	45	60	90	120	135	150	180	etc.
,, radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	

To convert degrees to radians, divide by 57·3;

$$i.e. \quad \theta^c = \frac{\theta^\circ}{57{\cdot}3}.$$

To convert radians to degrees, multiply by 57·3;

$$i.e.\quad \theta^\circ = 57{\cdot}3\theta^c.$$

From above, we have $\quad \text{arc} = r\theta; \quad \ldots\ldots(1)$

$$\therefore\ r = \frac{\text{arc}}{\theta}; \quad \ldots\ldots(2)$$

$$\therefore\ \theta = \frac{\text{arc}}{r}. \quad \ldots\ldots(3)$$

Suppose the point P moves once round the circumference of the circle (Fig. 69). Then the line OP will trace out 360° or 2π radians. If P moves round the circle n times per second, OP will trace out $2\pi n$ radians. This is termed the *angular velocity* of the point P, and is denoted by the Greek letter ω.

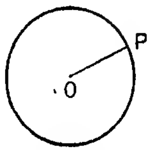


FIG. 69.

The *linear* velocity of P = circumference of circle × revs.p.s.

$$= 2\pi r n,$$

$$i.e.\quad v = \omega r.$$

v is generally expressed in feet per second; therefore r must be measured in feet.

EXAMPLES.

1. The radius of a circular arc is 12·75 cms. and the angle subtended at the centre 59·2°. Find the length of the arc.

$$\text{Arc} = r\theta \quad \ldots\ldots(1)$$
$$= 12{\cdot}75\theta;$$

but $\quad \theta = \frac{59{\cdot}2}{57{\cdot}3}$ radians;

$$\therefore\ \text{arc} = \frac{12{\cdot}75 \times 59{\cdot}2}{57{\cdot}3}$$
$$= \underline{13{\cdot}17 \text{ cm.}}$$

2. The length of a circular arc is 82·5 mm. and the angle subtended at the centre 37·92°. Find the radius.

$$r = \frac{\text{arc}}{\theta}; \quad \ldots\ldots(2)$$

$$\therefore\ r=\frac{82\cdot5}{\theta};$$

but $$\theta=\frac{37\cdot92}{57\cdot3}\text{ radians};$$

$$\therefore\ r=\frac{82\cdot5}{\frac{37\cdot92}{57\cdot3}}$$

$$=\frac{82\cdot5\times57\cdot3}{37\cdot92}$$

$$=\underline{124\cdot8\text{ mm.}}$$

3. The radius of a circle is 13·57′, and an arc is chosen whose length is 82·3′. What angle does it subtend at the centre (1) in radians, (2) in degrees?

$$\theta=\frac{\text{arc}}{r}$$

$$=\frac{82\cdot3}{13\cdot57}$$

$$=\underline{6\cdot064\text{ radians}}$$

$$=6\cdot064\times57\cdot3$$

$$=\underline{347\cdot5\text{ degrees.}}$$

4. A flywheel 12′ dia. makes 85 revolutions per minute. Find its angular velocity in radians per second, and its linear velocity in feet per second.

$$\text{Angular velocity}=2\pi n$$

$$=\frac{\overset{2\cdot833}{\cancel{2}\pi\times\cancel{85}}}{\underset{30}{\cancel{60}}},$$

i.e. $\omega=\underline{8\cdot9\text{ rad. per sec.}}$

$$\text{Linear velocity}=\omega r$$

$$=8\cdot9\times6,$$

i.e. v the velocity of a point on the rim $=\underline{53\cdot4\text{ ft. per sec.}}$

The linear velocity could have been obtained without finding the angular velocity. However, the method is important in mechanics.

Observe that the angular velocity is independent of the diameter of the wheel.

5. A belt passes over a pulley 3′ 6″ dia., and the angle of contact of the belt on the pulley is 190°. Determine the length of the curvilinear portion of the belt.

$$\text{Arc} = r\theta$$

$$= \frac{1{\cdot}75 \times 190}{57{\cdot}3},$$

i.e. curvilinear portion

$$= 5{\cdot}8' = 5'\ 10'' \text{ about.}$$

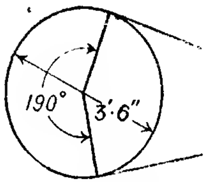


Fig. 70

6. A train runs on a curve 600′ rad. The length of the train is 450′. What angle does it subtend at the centre (1) in radians, (2) in degrees?

$$\theta = \frac{\text{arc}}{r}$$

$$= \frac{\overset{3}{\cancel{450}}}{\underset{4}{\cancel{600}}}$$

$$= \underline{0{\cdot}75 \text{ radians}}$$

$$= \tfrac{3}{4} \times 57{\cdot}3$$

$$= \frac{171{\cdot}9}{4}$$

$$= \underline{43^\circ \text{ about.}}$$

7. The diagram shows an indicator rig. Stroke of engine 3′, ON = 2′ 9″. Find Oa and the maximum value of θ for an indicator diagram 3′ long. The lever slides through a guide in the crosshead; find its minimum length.

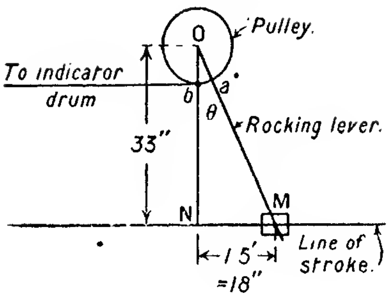


FIG. 71.

$$\frac{NM}{ON}=\tan\theta;$$

$$\therefore\ \tan\theta=\frac{\cancel{18}^{\,6}}{\cancel{33}_{\,11}}$$

$$=0{\cdot}5454;$$

$$\therefore\ \theta=\tan^{-1}0{\cdot}5454$$

$$=\underline{29^\circ\text{ about.}}$$

(See interpolation in the Appendix if greater accuracy is desired.)

Twice the arc *ab* through which the pulley rim moves must be the amount of cord wound on and off, *i.e.* neglecting stretch of cord

$$r=\frac{\text{arc}}{\theta}$$

$$=\frac{1{\cdot}5}{\frac{28{\cdot}5}{57{\cdot}3}}$$

$$=\frac{1{\cdot}5\times57{\cdot}3}{28{\cdot}5}$$

$$=3{\cdot}02'',$$

i.e. Oa the radius of the pulley is $\underline{3{\cdot}02''}$.

$$OM^2 = ON^2 + MN^2\,;$$
$$\therefore\ OM = \sqrt{ON^2 + NM^2}$$
$$= \sqrt{33^2 + 18^2}$$
$$= \sqrt{1089 + 324}$$
$$= \sqrt{1413}\,;$$

$\therefore$ minimum length of lever $= \underline{37{\cdot}59'' \text{ or } 3'\ 1{\cdot}59'', \text{ say } 3'\ 2''.}$

Belt Gearing. Two pulleys of radii r_1, r_2 are connected by an endless belt. If the centres are distant d from each other, find the length of belt necessary (*a*) open, (*b*) crossed.

(*a*)

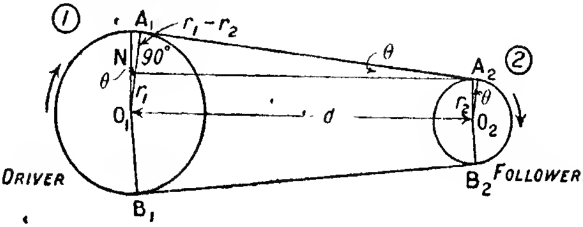


FIG. 72.

The radii r_1 and r_2 are at 90° to the belt, because it is assumed to be straight on both the tight and slack sides, and is therefore tangential to each pulley. Hence O_1A_1 is parallel to O_2A_2. Thus, $O_1O_2A_2N$ is a parallelogram.

Therefore $\qquad O_1O_2 = A_2N = d.$

Now $\qquad \dfrac{A_1A_2}{A_2N} = \dfrac{A_1A_2}{d} = \cos\theta\,;$

$$\therefore\ A_1A_2 = d\cos\theta = B_1B_2.$$

$$A_1\hat{O}_1B_1 = (\pi + 2\theta) \text{ radians}\,;$$

$\therefore$ length of curvilinear portion on $(1) = r_1(\pi + 2\theta)$.

$$A_2\hat{O}_2B_2 = (\pi - 2\theta) \text{ radians}\,;$$

$\therefore$ length of curvilinear portion on $(2) = r_2(\pi - 2\theta)$.

Total length = curved part on (1) + curved part on (2)
$+ A_1A_2 + B_1B_2$.

$$\therefore \quad L = r_1(\pi + 2\theta) + r_2(\pi - 2\theta) + 2d\cos\theta,$$

$$\text{i.e.} \quad L = \pi(r_1 + r_2) + 2\theta(r_1 - r_2) + 2d\cos\theta,$$

$$\text{or} \quad L = \frac{\pi}{2}(d_1 + d_2) + \theta(d_1 - d_2) + 2d\cos\theta$$

d_1 and d_2 being the diameters of the pulleys.

θ is obtained from the relation $\sin\theta = \dfrac{r_1 - r_2}{d}$,

$$\text{or} \quad \sin\theta = \frac{d_1 - d_2}{2d}.$$

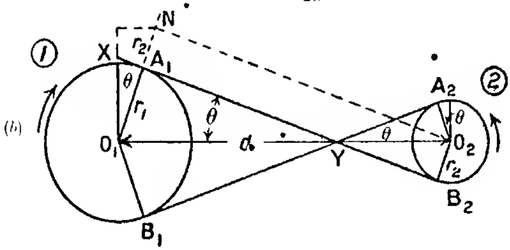


Fig. 73.

Assuming the tight and slack sides to be straight, the belt is tangential to both pulleys. O_1B_1 is parallel to O_2A_2, and O_1A_1 is parallel to O_2B_2. Hence $A_1\hat{O}_1B_1 = A_2\hat{O}_2B_2$ (larger angles). Draw O_2N parallel to A_1B_2 to meet O_1A_1 produced in N. Then $A_1N = r_2$, since $A_1NO_2B_2$ is a rectangle. Also $O_1\hat{O}_2N = \theta$, because the triangles XO_1Y and XO_1A_1 are equiangular,* and O_2N is parallel to YA_1

$$A_1B_2 = A_2B_1 = O_2N.$$

$$\frac{O_2N}{d} = \cos\theta;$$

$$\therefore \quad O_2N = d\cos\theta.$$

$$A_1\hat{O}_1B_1 = (\pi + 2\theta) \text{ radians},$$

$\therefore$ length of curvilinear portion on $(1) = r_1(\pi + 2\theta)$.

$$A_2\hat{O}_2B_2 = (\pi + 2\theta) \text{ radians},$$

$\therefore$ length of curvilinear portion on $(2) = r_2(\pi + 2\theta)$.

* Thus $O_1\hat{Y}A_1 = \theta$.

Total length = curved portion on (1) + curved portion on (2) $+ A_1B_2 + A_2B_1$;

$$\therefore\quad L = r_1(\pi + 2\theta) + r_2(\pi + 2\theta) + 2d\cos\theta,$$

$$i.e.\quad L = (\pi + 2\theta)(r_1 + r_2) + 2d\cos\theta,$$

$$\text{or}\quad L = \left(\frac{\pi}{2} + \theta\right)(d_1 + d_2) + 2d\cos\theta,$$

d_1 and d_2 being the diameters of the pulleys.

θ is obtained from the relation $\sin\theta = \dfrac{r_1 + r_2}{d}$,

$$\text{or}\quad \sin\theta = \frac{d_1 + d_2}{2d}.$$

Notice that the length of the straight portion of the belt is not the same in each case,* since the angles of lapping are larger when the belt is crossed, especially in the case of the smaller pulley. This would increase the tension at which the belt slipped. Crossed belts are used when the direction in which the second pulley (the follower) rotates has to be reversed. A certain amount of wear takes place when a belt is crossed due to rubbing action. There is an absence of such rubbing in an open belt, also it is slightly shorter.

If d is fixed and $(r_1 + r_2)$ remains constant, so also does $\sin\theta$, because $\sin\theta = \dfrac{r_1 + r_2}{d}$ for a crossed belt. Hence the length of a crossed belt remains constant, provided the sum of the radii of the pulleys remains constant. This condition must be satisfied in designing speed cones for a crossed belt. This does not hold when the belt is open; such a condition would make the length variable.

8. Two pulleys 3 ft. 6 ins. dia. and 2 ft. dia., centres 15 ft. apart, are connected by an endless belt. Find the length of belting necessary (1) open, (2) crossed.

(1) $L = \pi(r_1 + r_2) + 2\theta(r_1 - r_2) + 2d\cos\theta$

$= \pi(1{\cdot}75 + 1) + 2 \times 0{\cdot}05(1{\cdot}75 - 1) + 2 \times 15 \times 1$

$= 2{\cdot}75\pi + 0{\cdot}1 \times 0{\cdot}75 + 30$

$= 8{\cdot}64 + 30{\cdot}075$

$= 38{\cdot}72' = 38'\ 8{\cdot}6''$, say $38'\ 9''$.

$\sin\theta = \dfrac{r_1 - r_2}{d} = \dfrac{1{\cdot}75 - 1}{15} = \dfrac{0{\cdot}75}{15} = 0{\cdot}05.$

Now $\sin\theta = \theta$, when θ is small;

$\therefore\ \theta = 0{\cdot}05$ radian (see Appendix),

$\cos\theta = 1$ practically.

* The formula in each case is $d\cos\theta$.

If the belt was spliced for connecting the ends together, an additional amount would have to be allowed. The belting would have to be stretched over the pulleys before splicing to produce an initial tension when it was at rest.

This increases the resistance to slipping and also prevents centrifugal force raising the belt off the pulley.

(2) $L = (\pi + 2\theta)(r_1 + r_2) + 2d\cos\theta$

$= (\pi + 2 \times 0{\cdot}1833)(1{\cdot}75 + 1) + 2 \times 15 \times 0{\cdot}9832$

$= (3{\cdot}1416 + 0{\cdot}3666)\,2{\cdot}75 + 30 \times 0{\cdot}9832$

$= 3{\cdot}5082 \times 2{\cdot}75 + 29{\cdot}496$

$= 9{\cdot}647 + 29{\cdot}496$

$= 39{\cdot}14' = 39'\ 1{\cdot}7''$, say $39'\ 2''$,

which is about 5 ins. longer than the open belt.

$\sin\theta = \dfrac{r_1 + r_2}{d} = \dfrac{1{\cdot}75 + 1}{15} = \dfrac{2{\cdot}75}{15} = 0{\cdot}1833.$

Since θ is small,

$\theta = \sin\theta = 0{\cdot}1833^c = 10{\cdot}5°$,

$\cos 10{\cdot}5° = 0{\cdot}9832$.

(See approximations and interpolation in Appendix.)

The angles of lapping may be determined by a scale drawing, also the lengths of the straight portions. The lengths of the curved portions may then be calculated as shown in Example 5, page 70.

Conoids for Open Belt. Suppose two equal conoids, as shown (Fig. 74), are connected by an open belt. Find the radius at the centre of each conoid, so that the same length of belt will suffice at this point and at each end.

Length at each end

$= L_1 = \pi(r_1 + r_2) + 2\theta(r_1 - r_2) + 2d\cos\theta$ (see p. 73).

At the centre $\theta = 0$ and the radii are equal.

$\therefore$ L at centre $= \pi(2r) + 2d = L_1$;

$\therefore\ 2\pi r = L_1 - 2d$;

$\therefore\ r = \dfrac{L_1 - 2d}{2\pi}$,

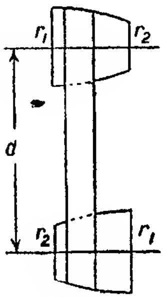


Fig. 74.

r being the radius at the centre of each conoid.

θ is found from the relation $\sin\theta = \dfrac{r_1 - r_2}{d}$.

Sector of a Circle. A sector of a circle is a portion included between two radii and an arc (Fig. 75).

Area of sector $=\frac{1}{2}r^2\theta$, θ being in radians; ......(1)

$$\left.\begin{aligned}\therefore\ r^2&=\frac{2A}{\theta};\\ \therefore\ r&=\sqrt{\frac{2A}{\theta}};\end{aligned}\right\}\ \ldots\ldots\ldots\ldots\ (2)$$

$$\therefore\ \theta=\frac{2A}{r^2}.\ \ldots\ldots\ldots\ldots\ (3)$$

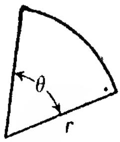


FIG. 75.

To take a particular case, let $\theta=2\pi$; then $\frac{1}{2}r^2\theta$ becomes πr^2, the area of a circle.

Portion of Ring.

Shaded area $=\frac{1}{2}r_1^2\theta-\frac{1}{2}r_2^2\theta$

$=\underline{\frac{1}{2}\theta(r_1^2-r_2^2)\text{ or }\frac{1}{2}(r_1-r_2)(r_1+r_2)\theta}$; .... (4)

$$\left.\begin{aligned}\therefore\ \theta&=\frac{2A}{(r_1^2-r_2^2)}\\ &=\frac{2A}{(r_1-r_2)(r_1+r_2)}.\end{aligned}\right\}\ \ldots\ldots\ (5)$$

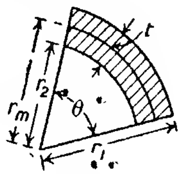


FIG. 76.

Notice that the shaded area may also be expressed thus:

$$\begin{aligned}A&=\tfrac{1}{2}\theta(r_1+r_2)(r_1-r_2)\\ &=\theta\left(\frac{r_1+r_2}{2}\right)\times(r_1-r_2)\\ &=\theta r_m t,\text{ where } r_m \text{ is the mean rad. and } t \text{ the thickness}\\ &=\underline{\text{mean arc}\times\text{thickness.}}\end{aligned}$$

EXAMPLES.

9. The angle of a circular sector is 85° and the radius 18·35 cm. Find the area in sq. cm. and sq. mm.

$$A = \tfrac{1}{2}r^2\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$
$$= \frac{18{\cdot}35^2 \times 85}{2 \times 57{\cdot}3}$$
$$= \underline{249{\cdot}8 \text{ sq. cm., say } 250,}$$
$$= \underline{24{,}980 \text{ sq. mm., say } 25{,}000.}$$

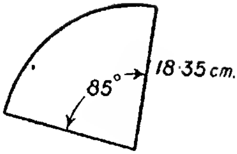


FIG. 77.

10. The area of a circular sector is 105·6 sq. ins. and the angle subtended at the centre 32°. Find the radius.

$$\left.\begin{aligned} r^2 &= \frac{2A}{\theta}; \\ \therefore\ r &= \sqrt{\frac{2A}{\theta}} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots \text{(2)}$$
$$= \sqrt{\frac{2 \times 105{\cdot}6}{\frac{32}{57{\cdot}3}}}$$
$$= \sqrt{\frac{\cancel{2} \times 105{\cdot}6 \times 57{\cdot}3}{\underset{16}{\cancel{32}}}}$$
$$= \underline{19{\cdot}45''.}$$

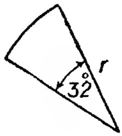


FIG. 78.

11. The area of a circular sector is 325 sq. mm. and the radius 2·12 cm. Find the angle in radians and degrees.

$$\theta = \frac{2A}{r^2} \quad \ldots\ldots\ldots\ldots \text{(3)}$$
$$= 2 \times \frac{325}{21{\cdot}2^2}$$
$$= \frac{650}{21{\cdot}2^2},$$

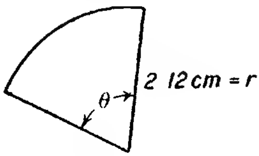


FIG. 79.

$$\textit{i.e.}\quad \theta = 1{\cdot}446 = \underline{1{\cdot}45 \text{ radians, say,}}$$
$$= 1{\cdot}446 \times 57{\cdot}3$$
$$= \underline{82{\cdot}88^\circ, \text{ say } 83^\circ.}$$

12. The length of a circular arc is 28·3″ and the radius 15·2″. Find the area of the sector.

$$A = \tfrac{1}{2}r^2\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$
$$= \frac{r}{2}(r\theta)$$
$$= \frac{r}{2} \times \text{arc}$$
$$= \frac{15{\cdot}2}{\cancel{2}} \times \overset{14{\cdot}15}{\cancel{28{\cdot}3}}$$
$$= \underline{215 \text{ sq. ins.}}$$

13. Find the area of a portion of a ring when $r_1 = 18{\cdot}3$ metres, $r_2 = 12{\cdot}7$ metres and $\theta = 71°$.

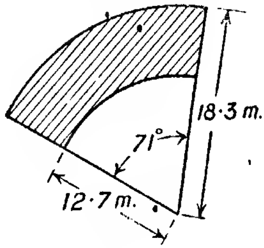


FIG. 80.

$$\left.\begin{aligned} A &= \tfrac{1}{2}\theta(r_1^2 - r_2^2); \\ \therefore\ A &= \tfrac{1}{2}\theta(r_1 - r_2)(r_1 + r_2) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (4)$$
$$= \frac{71}{2 \times 57{\cdot}3}(18{\cdot}3 - 12{\cdot}7)(18{\cdot}3 + 12{\cdot}7)$$
$$= \frac{71}{114{\cdot}6} \times 5{\cdot}6 \times 31$$
$$= \underline{107{\cdot}6 \text{ sq. metres.}}$$

Notice how convenient it is to use the factors of $(r_1^2 - r_2^2)$ in this case.

14. Fan blades have to be constructed from the following data: Area of blade 40 sq. ins., smaller radius 2 ins., larger radius 10 ins. Find the angle of a blade.

$$\left.\begin{aligned}\theta &= \frac{2A}{r_1^2 - r_2^2} \\ &= \frac{2A}{(r_1 - r_2)(r_1 + r_2)}\end{aligned}\right\} \ldots\ldots\ldots\ldots (5)$$

$$= \frac{2 \times 30}{(10 - 2)(10 + 2)}$$

$$= \frac{\overset{1}{\cancel{60}}}{\underset{1 \cdot 6}{\cancel{96}}} \text{ radian}$$

$$= \frac{57 \cdot 3 \times 1}{1 \cdot 6} \text{ degrees}$$

$$= \underline{35 \cdot 82^\circ, \text{ say } 36^\circ},$$

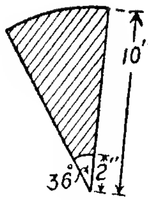


FIG. 81.

which is near enough for all practical purposes.

15. Three circles 4'', 4'' and 2'' dia. touch externally. Find the area included.

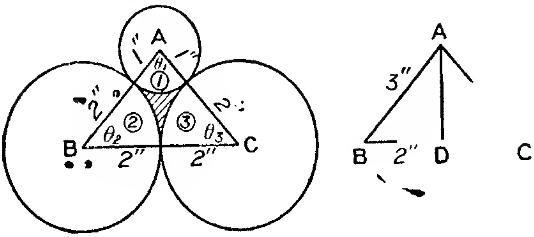


FIG. 82.

Area included = area of triangle ABC − area of 3 sectors.
ABC is an isosceles triangle, because AB = AC = 3''.
Drop a perpendicular AD on BC. Then BD = DC = 2''.

We have $\quad \dfrac{BD}{AB} = \dfrac{2}{3} = \cos B;$

$\therefore \cos B = 0 \cdot 6667$ correct to four places.

$\therefore B = \cos^{-1} 0 \cdot 6667$

$= \underline{48 \cdot 18^\circ}.$ (See Appendix.)

Since the triangle is isosceles,

$$C = B = 48{\cdot}18^\circ;$$

$$\therefore\ (B+C) = 2 \times 48{\cdot}18 = 96{\cdot}36^\circ.$$

$$A + B + C = 180^\circ;$$

$$\therefore\ A = 180^\circ - (B+C)$$
$$= 180^\circ - 96{\cdot}36$$
$$= \underline{83{\cdot}64^\circ}.$$

$AD = \sqrt{3^2 - 2^2}$, since ADB is a right-angled triangle,

$$= \sqrt{9-4}$$
$$= \sqrt{5}$$
$$= \underline{2{\cdot}236''}.$$

Area of triangle ABC $= AD \times BD$

$$= 2{\cdot}236 \times 2$$
$$= \underline{4{\cdot}472 \text{ sq. ins.}}$$

Area of sector (1) $= \frac{1}{2} r_1{}^2 \theta_1$

$$= \frac{1}{\cancel{2}} \times \frac{1^2 \times \overset{41{\cdot}82}{\cancel{83{\cdot}64}}}{57{\cdot}3}$$
$$= \frac{41{\cdot}82}{57{\cdot}3}.$$

Area of sectors (2) and (3) $= 2(\frac{1}{2} r_2{}^2 \theta_2)$, since they are both equal,

$$= \cancel{2}\left(\frac{1}{\cancel{2}} \times \frac{2^2 \times 48{\cdot}18}{57{\cdot}3}\right)$$
$$= \frac{4 \times 48{\cdot}18}{57{\cdot}3}$$
$$= \frac{192{\cdot}72}{57{\cdot}3}.$$

Combined area of sectors $= \dfrac{41{\cdot}82}{57{\cdot}3} + \dfrac{192{\cdot}7}{57{\cdot}3}$

$$= \frac{234{\cdot}52}{57{\cdot}3}$$
$$= \underline{4{\cdot}092 \text{ sq. ins.}}$$

Hence area included $= 4{\cdot}472 - 4{\cdot}092$

$$= \underline{0{\cdot}38 \text{ sq. in.}}$$

Notice that the areas of the sectors are not worked out separately. This simplifies the calculation.

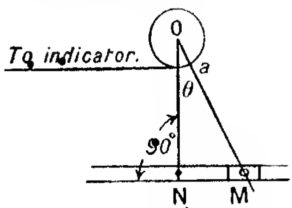
Examples to be Worked Out.

1. (*a*) $\theta = 37{\cdot}5°$.	Find θ radians.			
(*b*) ,, $= 136°$.	,, ,,			
(*c*) ,, $= 325°$.	,, ,,			
(*d*) ,, $= 0{\cdot}56$ radian.	Find θ degrees.			
(*e*) ,, $= 1{\cdot}32$,,	,, ,,			
(*f*) ,, $= 5{\cdot}41$,,	,, ,,			
(*g*) Arc $= 73{\cdot}2$ mms.	Radius $= 2{\cdot}5$ cms.	Find θ in degrees.		
(*h*) ,, $= 83{\cdot}7$ ins.	,, $= 1{\cdot}7$ ft.	,, ,,		
(*i*) ,, $= 32{\cdot}5$ ft.	,, $= 132{\cdot}9$ ins.	,, ,,		
(*j*) $\theta = 152°$.	,, $= 73{\cdot}2$ yds.	Find arc in feet.		
(*k*) ,, $= 93°$.	,, $= 35{\cdot}27$ metres.	,, ,, cms.		
(*l*) ,, $= 327{\cdot}6°$.	,, $= 21{\cdot}89$ kilometres.	,, ,, Km.		
(*m*) ,, $= 37{\cdot}2°$.	Arc $= 271{\cdot}2$ cms.	Find rad. in cms.		
(*n*) ,, $= 217°$.	,, $= 321{\cdot}5$ ins.	,, ,, ins.		
(*o*) ,, $= 315°$.	,, $= 123{\cdot}5$ ft.	,, ,, yds.		

2. A leather belt passes over a pulley 2′ 9″ dia., the angle of contact being 175°. Determine the length of the curvilinear portion of the belt.

3. A train runs on a curve 885′ rad., the length of the train being 476 ft. What angle does it subtend at the centre, (1) radians, (2) degrees?

4. The diagram shows an indicator rig. The stroke of the engine is 2′ 9″ and ON = 2′ 3″. Find θ and O*a* for an indicator diagram 3″ long, also the minimum length of the rocking lever.



5. Two pulleys whose diameters are 4′ 6″ and 2′ 8″ are connected by an endless belt. The centres are 24′ apart: find the length of belt (*a*) open, (*b*) crossed.

Make a scale drawing and use it to check the results. Measure the angles of contact.

6. Find the areas of the following sectors:

(a) $r = 92\cdot3''$.	$\theta = 13\cdot2°$.
(b) ,, $= 125\cdot7$ cms.	,, $= 183\cdot5°$.
(c) ,, $= 82\cdot1$ yds.	,, $= 279\cdot3°$.

7. The area of a sector of a circle is 125 sq. ins., and the angle subtended at the centre 93°. Find the radius.

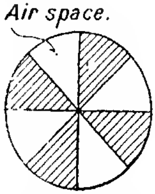
8. The radius of a circular sector is $3\cdot92'$ and the area 2571 sq. ins. Find the angle of the sector.

9. The development of a conical tent is a sector of a circle 16′ rad. The angle of the sector is 156°. Determine the area of canvas in sq. feet.

10. Three circles each 5″ dia. touch each other externally. Find the area included.

11. Three circles 4″, 4″ and $2\frac{1}{2}''$ dia. touch externally. Find, by the aid of an accurate drawing or otherwise, the area included.

12. A ventilator has to be constructed as shown. The total area of the air spaces is 100 sq. ins. Find the radius of the circle.



13. The crank of a horizontal engine is 2′ 6″ long. Find the angular and linear velocities of the crank pin, when the speed of the engine is 80 r.p.m., *i.e.* the crank makes 80 turns per minute.

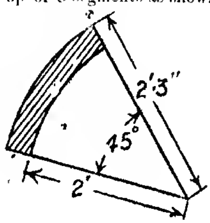
14. The pendulum of a clock oscillates through an angle of 10° on each side of the vertical, the length of arc through which it swings being 34 cms. Find the length of the pendulum in cms. and metres.

15. The area of a metal sector for a fan has to be 25 sq. ins. and the length of the blade 10″. Find the angle of the sector.

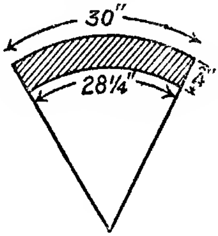
16. A fan blade has an area of 36 sq. ins.; the smaller rad. $= 2''$, larger 11″. Find the angle of the blade.

17. Find the area of a portion of a ring when $r_1 = 7\cdot69$ cms., $r_2 = 2\cdot82$ cms., $\theta = 91\cdot8°$.

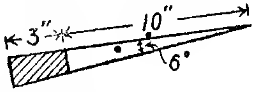
18. A wheel is built up of 8 segments as shown. Find the area of each segment.



19. The development of a clutch leather for a motor car is shown. Find its area.

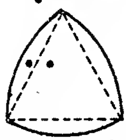


20. Find the cross-sectional area of the given commutator segment for a dynamo.



21. The commutator of an electric motor has 76 segments, each 25″ external dia. If the length of the commutator is 8″, find the surface area of each segment in sq. cms. and sq. decimetres. (The commutator is a cylinder 25″ dia., 8″ high. See p. 106.)

22. The cross-section of a tool employed for cutting square holes is shown. Find its area.



Equilateral triangle side = ¾″.
Centres of arcs at vertices of triangle.

23. The flywheel of a punching machine is 3′ dia. and rotates at 250 r.p.m. Find its angular velocity in radians per second.

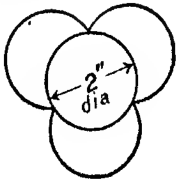
24. The peripheral speed of a flywheel is 4500 ft. per min. Find the angular velocity if the flywheel is 9 ft. 6 ins. dia.

25. Two equal conoids, diameters 2′ 9″ and 1′ 9″ and length 3′, are connected by an endless open belt. Find the radius at the centre so that the length of belt required there is the same as that at the ends. The centres are distant 15′. Draw the arrangement to a suitable scale.

26. Deduce a formula for the area included between three equal circles of radius a which touch externally.

27. Draw a circle 2″ dia. With three equidistant points on the circumference as centres, describe three circular arcs. Find (1) the length of each arc, (2) area enclosed by the three arcs, (3) radius and area of

the circle circumscribing the arcs, (4) ratio of radius of fundamental circle to circumscribing circle.



SEGMENT OF A CIRCLE.

SECTION 3.

If a chord AB be drawn anywhere in a circle, the two portions into which the circle is divided are called segments. If the chord is a diameter, *e.g.* CD, the segments are equal, but should the chord not be a diameter, the segments are unequal.

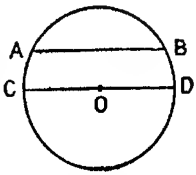


FIG. 83.

If AB is a diameter of the given circle, then any chord drawn at 90° to AB is bisected by AB. Thus, EF is bisected at D.

Since ODF is a right-angled triangle,

$$OD^2 + DF^2 = OF^2;$$

$$\therefore (r-h)^2 + \left(\frac{c}{2}\right)^2 = r^2 \quad (OD = AO - AD = r - h), \left(DF = \frac{c}{2}\right).$$

$$\therefore r^2 - 2rh + h^2 + \frac{c^2}{4} = r^2;$$

$$\therefore 2rh = \frac{c^2}{4} + h^2;$$

$$\therefore r = \frac{\frac{c^2}{4} + h^2}{2h}.$$

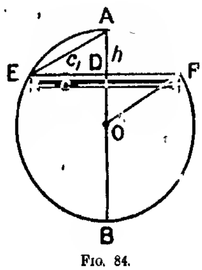


FIG. 84.

Moreover, if c the chord, and h the height of the segment are known, the radius of the circle from which the segment was cut may be determined.

The length of the arc EAF $= \dfrac{8c_1 - c}{3}$ approximately, c_1 being the chord of half the arc. For accurate work EOF should not exceed 180°. The area of the segment EAF $= \dfrac{h^3}{2c} + \dfrac{2}{3}ch$ approximately. This formula should only be used when the segment is less than a semicircle.

EXAMPLES.

1. Find the length of arc, area and radius of circle when $h = 4{\cdot}83$ cms., $c = 32{\cdot}9$ cms.

$$\text{Arc} = \frac{8c_1 - c}{3} = l.$$

$$c_1^2 = 4{\cdot}83^2 + 16{\cdot}45^2;$$

$$\therefore\ c_1 = \sqrt{4{\cdot}83^2 + 16{\cdot}45^2}$$

$$= \sqrt{23{\cdot}3 + 270{\cdot}5}$$

$$= \sqrt{293{\cdot}8}$$

$$= \underline{17{\cdot}14 \text{ cms.}}$$

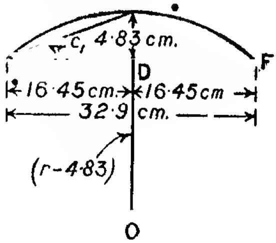


Fig. 85

Substituting, we have
$$\text{Arc} = \frac{8 \times 17{\cdot}14 - 32{\cdot}9}{3}$$

$$= \frac{137{\cdot}12 - 32{\cdot}9}{3}$$

$$= \frac{104{\cdot}22}{3}$$

$$= \underline{34{\cdot}74 \text{ cms.}}$$

$$\text{Area} = \frac{h^3}{2c} + \frac{2}{3}ch$$

$$= \frac{(4{\cdot}83)^3}{2 \times 32{\cdot}9} + \frac{2}{\cancel{3}} \times 32{\cdot}9 \times \overset{1{\cdot}61}{\cancel{4{\cdot}83}}$$

$$= \frac{(4{\cdot}83)^3}{65{\cdot}8} + 32{\cdot}9 \times 3{\cdot}22$$

$$= 1{\cdot}712 + 105{\cdot}9$$

$$= \underline{107{\cdot}6 \text{ sq. cms.}}$$

The first quantity is small compared with the second, because h is small compared with c. When this is the case, the term $\frac{h^3}{2c}$ may be left out, provided that no great accuracy is desired.

$$OF^2 = OD^2 + DF^2;$$
$$\therefore\ r^2 = (r - 4{\cdot}83)^2 + 16{\cdot}45^2;$$
$$\therefore\ \cancel{r^2} = \cancel{r^2} - 9{\cdot}66r + 4{\cdot}83^2 + 16{\cdot}45^2.$$
$$\therefore\ 9{\cdot}66r = 4{\cdot}83^2 + 16{\cdot}45^2;$$
$$\therefore\ r = \frac{4{\cdot}83^2 + 16{\cdot}45^2}{9{\cdot}66}$$
$$= \frac{23{\cdot}32 + 270{\cdot}5}{9{\cdot}66}$$
$$= \frac{293{\cdot}82}{9{\cdot}66}$$
$$= \underline{30{\cdot}42 \text{ cms.}}$$

2. A circle 10″ dia. is divided into two segments by a chord 2″ from the centre. Find the area of each segment and the angles subtended at the centre by the two arcs.

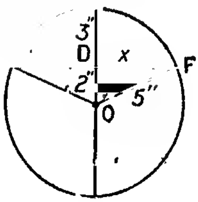


Fig. 86.

$$x^2 + 2^2 = 5^2;$$
$$\therefore\ x^2 = 5^2 - 2^2$$
$$= 25 - 4$$
$$= 21.$$
$$\therefore\ x = \sqrt{21}$$
$$= \underline{4{\cdot}58''},$$
$$\therefore\ c = 2 \times 4{\cdot}58 = \underline{9{\cdot}16''}.$$

$$\text{Area of upper segment} = \frac{h^3}{2c} + \frac{2}{3}ch$$
$$= \frac{3^3}{2 \times 9{\cdot}16} + \frac{2}{\cancel{3}} \times 9{\cdot}16 \times \cancel{3}$$
$$= \frac{\overset{13{\cdot}5}{\cancel{27}}}{\cancel{2} \times 9{\cdot}16} + 18{\cdot}32$$
$$= 1{\cdot}473 + 18{\cdot}32$$
$$= \underline{19{\cdot}79\cancel{3} \text{ sq. ins.}}$$

$$\text{Area of circle} = \frac{\pi \times 10^2}{4}$$
$$= 25\pi$$
$$= \underline{78 \cdot 54 \text{ sq. ins.}}$$

$$\therefore \text{ area of lower segment} = 78 \cdot 54 - 19 \cdot 79$$
$$= \underline{58 \cdot 75 \text{ sq. ins.}}$$

$$\cos \mathsf{DOF} = \tfrac{2}{5}$$
$$= 0 \cdot 4\,;$$
$$\therefore\ \hat{\mathsf{DOF}} = \cos^{-1} 0 \cdot 4 \quad \text{(see p. 35)}$$
$$= \underline{66 \cdot 5^\circ \text{ about.}}$$

$$\therefore\ 2\hat{\mathsf{DOF}} = \underline{133^\circ = \text{angle subtended by smaller arc,}}$$
$$360^\circ - 133^\circ = \underline{227^\circ = \text{angle subtended by larger arc.}}$$

3. A boiler is 7′ dia. and has two furnace tubes 2′ 6″ dia. situated in the water space. The water is 4′ 9″ from the bottom; find the area of the steam space and the net area of the water space.

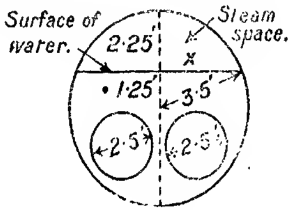


FIG. 87.

$$\text{Area of steam space} = \frac{h^3}{2c} + \frac{2}{3}ch.$$

Now $$h = 7 - 4 \cdot 75 = \underline{2 \cdot 25'}.$$

And $$x^2 + 1 \cdot 25^2 = 3 \cdot 5^2\,;$$
$$\therefore\ x^2 = 3 \cdot 5^2 - 1 \cdot 25^2$$
$$= 12 \cdot 25 - 1 \cdot 563$$
$$= 10 \cdot 687, \text{ say } 10 \cdot 69\,;$$
$$\therefore\ x = \sqrt{10 \cdot 69}$$
$$= \underline{3 \cdot 27'}. \quad \therefore\ c = 2 \times 3 \cdot 27 = \underline{6 \cdot 54'}.$$

$$\therefore\ \mathsf{A} = \frac{(2 \cdot 25)^3}{2 \times 6 \cdot 54} + \frac{2}{\cancel{3}} \times 6 \cdot 54 \times \overset{0 \cdot 75}{\cancel{2 \cdot 25}}$$

$$= \frac{2 \cdot 25^3}{13 \cdot 08} + 6 \cdot 54 \times 1 \cdot 5$$

$$= 0 \cdot 8712 + 9 \cdot 81$$

$$= \underline{10 \cdot 6812 \text{ sq. ft.}}$$

Net area of water space = area of large circle − (area of 2 flues + area of steam space)

$$= \frac{\pi \times 7^2}{4} - \left(\frac{2\pi \times 2 \cdot 5^2}{4} + 10 \cdot 68\right)$$

$$= \frac{\pi}{4}\{49 - 12 \cdot 5\} - 10 \cdot 68$$

$$= \frac{\pi}{\cancel{4}} \times \overset{9 \cdot 125}{\cancel{36 \cdot 5}} - 10 \cdot 68$$

$$= 28 \cdot 67 - 10 \cdot 68$$

$$= \underline{17 \cdot 99, \text{ say } 18 \text{ sq. ft.}}$$

In all cases the crowns of the furnace tubes are submerged, otherwise they would be burnt.

Examples to be Worked Out.

1. Find the length of arc and the radius of the original circle in each of the following cases:

(*a*) $h = 5''$, $c = 36''$. (*b*) $h = 2 \cdot 1$ cms., $c = 9 \cdot 4$ cms.

(*c*) $h = 11 \cdot 51$ mms., $c = 89 \cdot 5$ mms. (*d*) $h = 2 \cdot 1'$, $c = 6 \cdot 92'$.

2. Find the areas of the following segments and the angles subtended at the centres by the arcs.

(*a*) $h = 5''$, $c = 16''$, (*b*) $h = 19 \cdot 0$ cms., $c = 40$ cms., (*c*) $h = 5 \cdot 6'$, $c = 8'$.

(The angles may be calculated or obtained from a scale drawing.)

3. A Lancashire boiler 8′ dia. contains water to a level 5′ 10″ from the bottom. Find the cross-sectional area of the steam space, *i.e.* the space above the water level. (The boiler is a hollow cylinder 8′ internal dia. with its axis horizontal.)

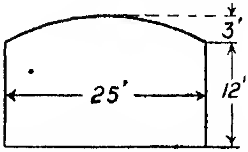
4. A Lancashire boiler 8′ dia. has two furnace tubes 3′ 3″ dia. in the water space. The water level is 5′ 8″ from the bottom. Find the area of the steam space and the net area of the water space. (The furnace tubes are cylindrical, their axes being parallel to the axis of the boiler shell.)

5. The front end plate of a cylindrical boiler is 12′ dia. and is made in three sections of equal depth. Find the weight of each section, if the plate is 1½″ thick. 1 sq. ft. of ⅛″ steel plate = 5·1 lbs.

6. The formula for the area of a circular segment, viz. $\frac{h^3}{2c}+\frac{2}{3}ch$, is only approximate. Calculate the percentage error if it is used to find the area of a semicircle whose radius is r.

7. The area of a segment of a circle is 893 sq. mms., and the ratio of the chord to the height 5 : 1. Find the chord, height and radius of the original circle.

8. Find the cross-sectional area of the following segmental arch :



9. A circle 8″ dia. is divided into two segments by a chord $2\frac{1}{2}$″ from the centre. Find the area of each segment.

CHAPTER IV.

THE ELLIPSE.*

CD is the major axis $= 2a$.

AB is the minor axis $= 2b$.

OC $=$ OD $=$ semi-major axis $= a$.

OA $=$ OB $=$ semi-minor axis $= b$.

Perimeter of ellipse
$= \pi(a+b)$ approximately.†

Area of ellipse $= \pi ab$.

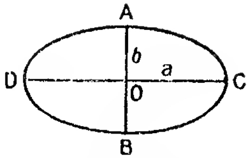


FIG. 88.

When the major and minor axes are equal, $a = b$, and the area is πa^2, *i.e.* the ellipse becomes a circle.

If $m_1 =$ major axis and $m_2 =$ minor axis,

the area $A = 0{\cdot}7854\, m_1 m_2$.

When $m_1 = m_2$ we have $A = 0{\cdot}7854\, m_1^2$, *i.e.* the area of a circle whose diameter is m_1.

Elliptical Ring.

OB $= a_1$, OA $= a_2$.
OD $= b_1$, OC $= b_2$.

The area included between two ellipses, whether concentric or eccentric, provided one is wholly inside the other, is $\pi(a_1 b_1 - a_2 b_2)$.

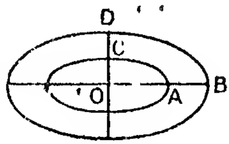


FIG. 89.

In the case shown, the ellipses are concentric (Fig. 89).

* For definition of the ellipse, see books on geometry.

† The perimeter of an ellipse can only be determined approximately. Closer approximations are

$$(1)\ 2\pi\sqrt{\frac{a^2+b^2}{2}}. \qquad (2)\ \pi\sqrt{\frac{m_1^2+m_2^2}{2} - \frac{(m_1-m_2)^2}{8{\cdot}8}}.$$

The latter formula is the most accurate of the three.

EXAMPLES.

1. An ellipse has a major axis of 10″ and a minor axis of 7·5″. Find its perimeter and area.

$$\begin{aligned}\text{Perimeter} &= \pi(a+b)\\ &= \pi(5+3{\cdot}75)\\ &= 8{\cdot}75\pi\\ &= \underline{27{\cdot}5'' \text{ approx.}}\end{aligned}$$

$$\begin{aligned}\text{Area} &= \pi ab\\ &= \pi \times 5 \times 3{\cdot}75\\ &= \underline{58{\cdot}91 \text{ sq. ins.}}\end{aligned}$$

2. Find the dia. and area of a circle whose perimeter is equal to that of the ellipse in Example (1).

$$\begin{aligned}\text{Perimeter of circle} &= \text{perimeter of ellipse};\\ \therefore\ \cancel{\pi}d &= \cancel{\pi}(a+b);\\ \therefore\ d &= a+b\\ &= 5+3{\cdot}75,\\ \textit{i.e.}\quad \text{dia. of circle} &= \underline{8{\cdot}75''}.\end{aligned}$$

$$\begin{aligned}\text{Area of circle} &= 0{\cdot}7854d^2\\ &= 0{\cdot}7854 \times 8{\cdot}75^2\\ &= \underline{60{\cdot}13 \text{ sq. ins.}}\end{aligned}$$

On comparing the two areas it will be seen that the area of the circle is greater than the area of the ellipse. The figure of greatest area, for a given perimeter, is a circle.

3. An elliptical steel plate weighs 25 lbs. The major and minor axes of the ellipse are 15″ and 11″. Find its thickness if 1 sq. ft. of $\frac{1}{8}''$ steel plate = 5·1 lbs.

Weight = area in sq. ft. × wt. of 1 sq. ft.

Now, the wt. of 1 sq. ft. of $\frac{1}{8}''$ plate = 5·1 lbs.;

∴ „ „ 1″ „ = 5·1 × 8 = 40·8 lbs.;

∴ „ „ plate t'' thick = 40·8t lbs.

Hence, $W = A \times 40{\cdot}8t$, A being in sq. ft. and t in inches.

$$\therefore\ t=\frac{W}{A\times 40{\cdot}8}$$

$$=\frac{25}{\dfrac{0{\cdot}7854\times 15\times 11\times 40{\cdot}8}{144}}$$

$$=\frac{\overset{5}{\cancel{25}}\times\overset{\overset{6}{\cancel{48}}}{\cancel{144}}}{0{\cdot}7854\times\underset{3}{\cancel{15}}\times 11\times\underset{5{\cdot}1}{\cancel{40{\cdot}8}}}$$

$$=\frac{30}{0{\cdot}7854\times 56{\cdot}1}$$

$$=\underline{\underline{0{\cdot}681''}}.$$

Examples to be Worked Out.

1. Calculate the perimeter and area of each of the following ellipses:

(*a*)	Major axis or $2a$	$=32{\cdot}3$ ins.	Minor axis or $2b$	$=17{\cdot}6$ ins.
(*b*)		$=25{\cdot}1$ cms.		$=12{\cdot}8$ cms.
(*c*)	,,	$=832$ mms.	,,	$=765$ mms.
(*d*)	,,	$=10{\cdot}2$ ft.	,,	$=52$ ins.

2. The area of an ellipse is $59{\cdot}72$ sq. metres and the ratio of the major to the minor axis is $1{\cdot}32:1$. Find each axis, and the perimeter.

3. An ellipse has a major axis of 8″ and a minor axis of 6″. Find its area. Find also the dia. and area of a circle whose perimeter is equal to the perimeter of the ellipse.

4. The perimeter of an ellipse is $153{\cdot}5''$ and the ratio of the axes $5:4$. Determine each axis and the area.

5. The pitch line of an elliptical toothed wheel has a major axis of 20″ and a minor axis of 12″. There are 45 teeth: find the pitch. (See p. 59.)

6. The number of rivets used in connection with a mud-hole door is 20, and the axes of the ellipse are $17\frac{1}{2}''$ and $13\frac{1}{2}''$. Determine the pitch of the rivets.

7. An elliptical plate of steel has a major axis of 13″ and a minor axis of 10″. It is $\frac{5}{8}''$ thick. Find its weight, if 1 sq. ft. of $\frac{1}{8}''$ plate $=5{\cdot}1$ lbs.

8. An elliptical plate has a major axis of $18{\cdot}9''$. Its weight is $35{\cdot}5$ lbs. and thickness $\frac{3}{8}''$. Find the minor axis of the ellipse. 1 sq. ft. $\frac{1}{8}''$ steel plate $=5{\cdot}1$ lbs.

9. An elliptical plate weighs 20 lbs., the major and minor axes being 14 ins. and 10 ins. Find its thickness. 1 sq. ft. $\frac{1}{8}''$ steel plate $=5{\cdot}1$ lbs.

10. Find the diameter of a circle equal in area to an ellipse whose major axis is 20″ and minor axis 10″.

CHAPTER V.

THE CUBE.

A CUBE or regular hexahedron (six-faced solid) is a solid figure bounded by six equal faces, each face being a square (Fig. 90).

Volume = cross-sectional area × length
$= a^2 \times a$
$= \underline{a^3}.$

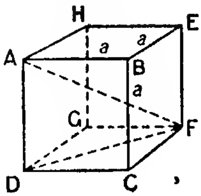


FIG. 90.

Since it has six faces and the area of each face is a^2, the total superficial area $= \underline{6a^2}$.

A cube has four equal diagonals The diagonals join opposite corners, and all of them meet in a point. This point is the centre of the solid.

AF is one diagonal; BG, CH and DE are the three others.

To find the length of a diagonal.

CDF is a right-angled triangled because $\hat{C} = 90°$;

$$\therefore\ DC^2 + CF^2 = DF^2;$$
$$\therefore\ a^2 + a^2 = DF^2;$$
$$\therefore\ 2a^2 = DF^2.$$

ADF is a right-angled triangle because $\hat{D} = 90°$;

$$\therefore\ DF^2 + AD^2 = AF^2;$$
$$\therefore\ 2a^2 + a^2 = AF^2;$$
$$\therefore\ AF^2 = 3a^2;$$
$$\therefore\ AF = \underline{a\sqrt{3} = \text{length of a diagonal.}}$$

If we imagine the four diagonals to be drawn, the cube is divided into six equal pyramids, each of which has a square base, *i.e.* one of the faces of the cube. The apex of each pyramid is the point of concurrency of the diagonals,* so that the altitude is $\frac{1}{2}$ the edge of the cube.

Since there are six pyramids, the volume of each is $\frac{1}{6}$ the volume of the cube.

$$\text{Now vol. of cube} = a^3,\ a \text{ being the edge};$$

$$\therefore \text{ vol. of 1 pyramid} = \frac{a^3}{6}$$

$$= \frac{a^2}{3} \times \frac{a}{2};$$

i.e. vol. of pyramid $= \frac{1}{3}$ area of base × altitude.

This formula is true for all pyramids, no matter what the shape of the base may be.

The rectangular prism. A rectangular prism is a solid having six faces, each face being a rectangle, *i.e.* it is an irregular hexahedron (Fig. 91).

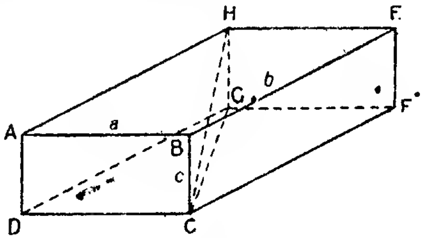


Fig. 91.

The opposite faces are parallel, and any cross-section by a plane parallel to one of the faces is identical to that face.

The solid has four equal diagonals, all of which meet in a point. This point is the centre of the solid. HC is one diagonal; BG, AF and DE are the three others.

Volume of prism = cross-sectional area × length or height

$= ac \times b =$ length × breadth × thickness

$= abc.$

* The centre of the cube.

When $a=c$, the face ABCD is a square, and the solid is a square prism. Its volume $=\underline{a^2b}$.

When $a=b=c$, all the faces are equal squares; hence the solid is a cube whose volume is $\underline{a^3}$, *i.e.* a cube is a particular case of a rectangular prism.

Surface of prism

$$\begin{aligned}&= 2 \times \text{area of ABCD} + 2 \times \text{area of BECF} + 2 \times \text{area of ABEH}\\ &= 2 \times ac + 2 \times bc + 2 \times ab\\ &= 2(ab + bc + ca).\end{aligned}$$

To find the length of a diagonal.

FGC is a right-angled triangle because $\hat{F} = 90°$;

$$\therefore\ FG^2 + FC^2 = CG^2;$$
$$\therefore\ a^2 + b^2 = CG^2.$$

HGC is a right-angled triangle because $\hat{G} = 90°$;

$$\therefore\ CG^2 + GH^2 = CH^2;$$
$$\therefore\ a^2 + b^2 + c^2 = CH^2 \text{ (by substitution)};$$
$$\therefore\ CH = \sqrt{a^2 + b^2 + c^2};$$

i.e. the length of any diagonal $= \underline{\sqrt{a^2 + b^2 + c^2}}$.

For a cube $a=b=c$, hence diagonal $= \sqrt{a^2 + a^2 + a^2}$

$$= \sqrt{3a^2}$$
$$= a\sqrt{3}, \text{ as shown before.}$$

The centre of the prism, *i.e.* the point of concurrency of the four diagonals, is equidistant from the eight corners. Consequently a sphere having this point as centre and half a diagonal as radius will pass through each corner, *i.e.* every corner will be on its surface. This sphere is called the circumscribing sphere. Its radius is $\frac{1}{2}\sqrt{a^2 + b^2 + c^2}$. For a cube the radius is evidently $a \times \dfrac{\sqrt{3}}{2}$.

Prisms in general. If a solid rests on the horizontal plane, and all sections by planes parallel to H.P. are identical, the solid is a prism. The base of a prism may be any rectilinear figure regular or irregular. When the faces* of the prism are at 90° to H.P., the prism is a right prism. Moreover, the

* Not the end faces.

axis or imaginary line joining the centre of the base to the centre of the top is at 90° to the base.

When the faces are inclined obliquely to H.P. the prism is an oblique prism. Obviously its axis is oblique.

The volume of any prism = area of cross-section × altitude.

EXAMPLES OF PRISMS.

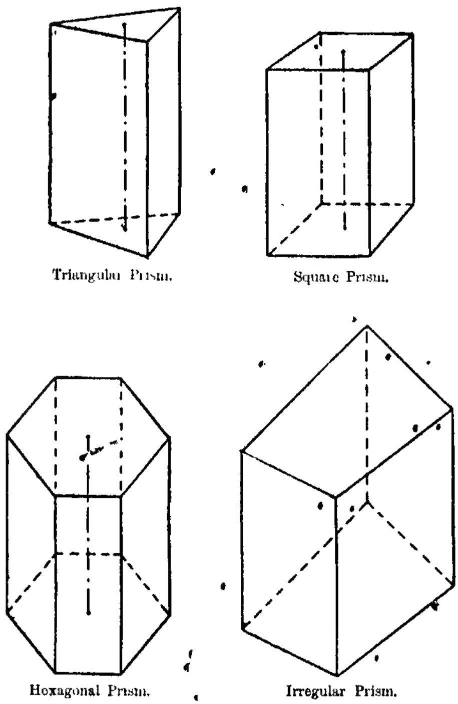


Fig. 42.

EXAMPLES.

1. A cube has an edge of 9·54″. Determine its volume in cu. ins. and cu. ft., and its surface in sq. ins. and sq. ft.

$$\text{Volume} = a^3$$
$$= (9{\cdot}54)^3$$
$$= \underline{868 \text{ cubic ins.}}$$

Now $\quad 12 \text{ ins.} = 1 \text{ ft.};$

$$\therefore\ 12^2 \text{ ins.}^2 = 1 \text{ sq. ft.};$$
$$\therefore\ 144 \text{ sq. ins.} = 1 \text{ sq. ft.};$$
$$\therefore\ 12^3 \text{ ins.}^3 = 1 \text{ cu. ft.};$$
$$\therefore\ 1728 \text{ cu. ins.} = 1 \text{ cu. ft.};$$
$$\therefore\ \text{volume} = \tfrac{868}{1728}$$
$$= \underline{0{\cdot}5 \text{ cu. ft. about.}}$$
$$\text{Surface} = 6a^2$$
$$= 6 \times (9{\cdot}54)^2$$
$$= \underline{546 \text{ sq. ins.}}$$
$$\text{Surface} = \tfrac{546}{144}$$
$$= \underline{3{\cdot}79 \text{ sq. ft.}}$$

2. A cube has a volume of 100 cu. ins. Find its edge.

$$V = a^3;$$
$$\therefore\ a = \sqrt[3]{V}$$
$$= \sqrt[3]{100},$$

i.e. the edge $= \underline{4{\cdot}64''}$.

3. The edge of a cube is 14·3″. Determine the length of a diagonal.

$$\text{Length of diagonal} = a\sqrt{3}$$
$$= 14{\cdot}3 \times 1{\cdot}732$$
$$= \underline{24{\cdot}77''}.*$$

4. 1 cu. ft. of water weighs 62·5 lbs. Find the pressure per sq. in. on the bottom of a tank 3′ × 2′ × 4′ deep.

$$\text{Wt. of water in tank} = \text{vol.} \times \text{wt. of 1 cu. ft.}$$
$$= 3 \times 2 \times 4 \times 62{\cdot}5$$
$$= \underline{24 \times 62{\cdot}5 \text{ lbs.}}$$

* The radius of the circumscribing sphere is $\frac{24{\cdot}77}{2} = \underline{12{\cdot}39''}$.

$$\text{Pressure per square inch} = \frac{\text{wt. in lbs.}}{\text{area of bottom in sq. ins.}}$$

$$= \frac{\cancel{24} \times 62 \cdot 5}{3 \times 2 \times \cancel{144}_{6}}$$

$$= \frac{62 \cdot 5}{36}$$

$$= \underline{1 \cdot 74 \text{ lbs.}}$$

5. 1 foot $= 3 \cdot 048$ decimetres. Find the number of cubic dm. in 1 cubic ft.

$$1 \text{ ft.} = 3 \cdot 048 \text{ dm.}$$

$$\therefore (1 \text{ ft.})^3 = (3 \cdot 048 \text{ dm.})^3;$$

$$\therefore 1 \text{ cubic ft.} = 3 \cdot 048^3 \text{ cubic dm.}$$

$$= \underline{28 \cdot 44 \text{ cubic dm.}}$$

6. A cube of steel $6 \cdot 5''$ edge is hammered—after being heated—into a square prism, an edge of whose base is $3 \cdot 25''$. Find the height of the prism.

$$\text{Volume of cube} = 6 \cdot 5^3$$

$$= \underline{274 \cdot 6 \text{ cu. ins.}}$$

Now $\quad \text{vol. of prism} = \text{vol. of cube};$

$$\therefore h \times 3 \cdot 25^2 = 274 \cdot 6;$$

$$\therefore h = \frac{274 \cdot 6}{3 \cdot 25^2}$$

$$= \underline{26''}.$$

7. The ratio of the dimensions of a rectangular prism is $1 : 2 : 3$, and its volume is 840 cu. ins. Determine its dimensions.

$$\text{Volume} = abc.$$

Now $\quad \frac{a}{b} = \frac{1}{2};$

$$\therefore \underline{b = 2a},$$

$$\frac{a}{c} = \frac{1}{3};$$

$$\therefore \underline{c = 3a}.$$

Hence $\quad abc = a \times 2a \times 3a$

$$= \underline{6a^3}.$$

But
$$6a^3 = 840;$$
$$\therefore a^3 = \frac{\overset{140}{\cancel{840}}}{\cancel{6}};$$
$$\therefore a = \sqrt[3]{140};$$
$$\left.\begin{aligned} \therefore a &= \underline{5{\cdot}19''}, \\ b = 2a &= \underline{10{\cdot}38''}, \\ c = 3a &= \underline{15{\cdot}57''}. \end{aligned}\right\}$$

The result can be checked by finding the product of 5·19″, 10·38″, 15·57″. This ought to give 840 cubic ins. In this case the product is 840·7 cu. ins., which is accurate enough.

8. A steel lever $\frac{1}{2}''$ sq. section is 3′ 3″ long. Find its weight if 1 cu. in. steel weighs 0·28 lb.

$$\begin{aligned} \text{Weight} &= \text{vol. in cu. ins.} \times \text{wt. of 1 cu. in.} \\ &= 39 \times 0{\cdot}5^2 \times 0{\cdot}28 \\ &= \frac{39 \times \overset{0{\cdot}07}{\cancel{0{\cdot}28}}}{\cancel{4}} \\ &= \underline{2{\cdot}73 \text{ lbs.}} \end{aligned}$$

9. A vessel displaces 22,000 tons of fresh water. How many cubic ft. of salt water will it displace? 1 cu. ft. salt water = 64 lbs.

$$\text{Weight of water displaced}^* = 22{,}000 \times 2240 \text{ lbs.}$$
$$\begin{aligned} \text{Vol. of salt water displaced} &= \frac{22{,}000 \times 2240}{\text{lbs. per cu. ft.}} \\ &= \frac{22{,}000 \times 2240}{64} \\ &= \underline{770{,}000 \text{ cu. ft.}} \end{aligned}$$

10. The extension of a helical steel spring is proportional to the load. A piece of metal 3″ × 3″ × 12″ produces an extension of 3·95″. 14 lbs. produces an extension of 1·83″. Find the weight of 1 cu. in. of the metal.

* A ship displaces the same weight of fresh water as it does salt water. The weight of water displaced is equal to the weight or tonnage of the ship.

$$\text{Weight of metal} = \text{vol. in cu. ins.} \times \text{wt. of 1 cu. in.}$$
$$= 3 \times 3 \times 12 \times w$$
$$= \underline{108w \text{ lbs.}}$$

14 lbs. produces an extension of 1·83″.

Since the extension is proportional to the load,

1 lb. produces an extension of $\frac{1 \cdot 83''}{14}$;

$\therefore$ 108w lbs " " " $\frac{1 \cdot 83 \times 108w}{14}$.

Hence $$\frac{1 \cdot 83 \times 108w}{14} = 3 \cdot 95;$$

$$\therefore w = \frac{3 \cdot 95 \times 14}{1 \cdot 83 \times 108},$$

i.e. the wt. of 1 cu. in. $= \underline{0 \cdot 2798, \text{ say } 0 \cdot 28 \text{ lb.}}$

Now 1 cu. in. steel $= 0 \cdot 28$ lb., so that we may conclude that the metal is steel.

11. An electric accumulator $10'' \times 14'' \times 10''$ high contains sulphuric acid to a depth of 7″. If the specific gravity of the acid is 1·195, find its weight. 1 cu. ft. water $= 62 \cdot 5$ lbs.

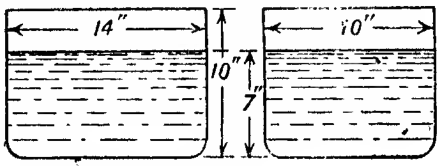


FIG. 93.

$$\text{Volume of acid} = 10 \times 14 \times 7$$
$$= 980 \text{ cu. ins.}$$
$$= \frac{980}{1728} \text{ cu. ft., since there are 1728 cu. ins. in 1 cu. ft.}$$

$$\text{Specific gravity} = \frac{\text{wt. of a volume } v \text{ of a substance}}{\text{wt. of a volume } v \text{ of water}},$$

$$\therefore \text{wt. of subs.} = \text{sp. gr.} \times \text{wt. of same vol. of water}$$
$$= 1 \cdot 195 \times \frac{980}{1728} \times 62 \cdot 5,$$

i.e wt. of acid $= \underline{42 \cdot 4 \text{ lbs.}}$

When an electric accumulator is fully charged the specific gravity of the acid is greater than when partially charged. During charge the specific gravity gradually rises, owing to the proportion of water becoming less, and that of acid greater. The specific gravity above, viz. 1·195, represents the condition of the acid when the accumulator is partially charged.

12. A wedge, whose cross-section is an isosceles triangle, base 1·5″ and altitude 4″, is 6·5″ long. Find its volume.

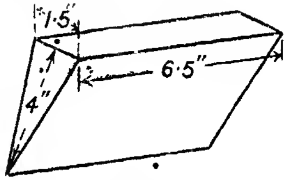


FIG. 94.

The wedge is a triangular prism.

$$\text{Volume} = \text{area of cross-section} \times \text{length}$$
$$= \frac{4 \times 1{\cdot}5}{2} \times 6{\cdot}5$$
$$= 3 \times 6{\cdot}5$$
$$= \underline{19{\cdot}5 \text{ cubic ins.}}$$

13. A steel knife edge whose cross-section is an equilateral triangle $\frac{1}{2}''$ side is $1\frac{1}{2}''$ long. Find its volume and weight if 1 cu. in. steel $= 0{\cdot}28$ lb.

$$\text{Volume} = \text{cross-sectional area} \times \text{altitude or length}$$
$$= 0{\cdot}433a^2l$$
$$= 0{\cdot}433 \times \left(\frac{1}{2}\right)^2 \times 1\tfrac{1}{2}$$
$$= 0{\cdot}433 \times \frac{1}{4} \times \frac{3}{2}$$
$$= \frac{1{\cdot}299}{8}$$
$$= \underline{0{\cdot}1624 \text{ cu. in. correct to the fourth place.}}$$

$$\text{Weight} = \text{vol. in cu. in.} \times \text{wt. of 1 cu. in.}$$
$$= 0{\cdot}1624 \times 0{\cdot}28$$
$$= \underline{0{\cdot}0455 \text{ lb.}}$$

14. A cotter 15″ long is 1″ wide at the thin end. It has a taper of 1 in 20. Find its volume if it is $\frac{3}{4}$″ thick throughout its length.

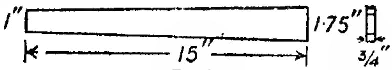


Fig. 95.

Increase of width $= \frac{15}{20}$ (see taper in the Appendix)

$= \frac{3}{4}''$;

$\therefore$ width at thick end $= 1 + \frac{3}{4}$

$= \underline{1.75''}$.

Volume = area of cross-section × thickness

= mean width × length × thickness

$= \left(\frac{1 + 1.75}{2}\right) 15 \times 0.75$

$= \frac{2.75}{2} \times 15 \times 0.75$

$= \underline{15.46 \text{ cu. ins.}}$

The weight of the cotter in the above example

$= 15.46 \times 0.28$

$= \underline{4.33 \text{ lbs.}}$

(assuming it to be made of steel weighing 0·28 lb. per cu. in.)

15. A steel plate (rhombus with sides $6\frac{1}{2}''$ long and base angle 70°) weighs 6 lbs. Find its thickness if 1 cu. in. = 0·283 lb.

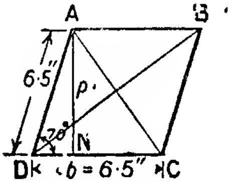


Fig. 96.

Determine the area by drawing to scale, or proceed as follows:

Area $= p \times b$.

Now $$\frac{p}{6\cdot5}=\sin 70°;$$

$$\therefore\ p=6\cdot5\sin 70°$$
$$=6\cdot5\times0\cdot9397;$$
$$\therefore\ \text{area}=6\cdot5\times0\cdot9397\times6\cdot5$$
$$=\underline{39\cdot7\text{ sq. ins.}}$$

Volume = area × thickness = At.

Weight = vol. in cu. ins. × wt. of 1 cu. in.

$$\therefore\ W=At\times0\cdot283;$$
$$\therefore\ t=\frac{W}{A\times0\cdot283}$$
$$=\frac{6}{39\cdot7\times0\cdot283}$$
$$=\underline{0\cdot534''}.$$

Examples to be Worked Out.

1. The edge of a cube is 7·3″; determine its volume. Draw a plan and elevation to scale.

2. The volume of a cube is 973 cu. ins.; determine its edge. Draw a plan and elevation to scale.

3. 1 in. = 2·54 cms. How many cu. cms. = 1 cu. in.

4. The edge of a cube is 12·3″; determine the length of the diagonal. Draw plan and elevation and show diagonal.

5. A cube has a volume of 1932 cu. mms.; determine the length of its diagonal. Draw a plan and elevation and show diagonal.

6 The edge of a cube is 72·5 cms; find the radius of its circumscribing sphere. Draw a plan and elevation to scale.

7. Determine the surfaces of the cubes in (1) and (6).

8. A cube has a superficial area of 950 sq. ins.; calculate its edge.

9. One cu. ft. of cast iron weighs about 450 lbs. and 1 cu. ft. of water 62·5 lbs.; make a scale drawing showing the relative weights.

10. One cu. ft. of water weighs 62·5 lbs.; determine the pressure per sq. in. on the bottom, if the cube was contained in a tank 1′ × 1′ × 1′.

11. The pressure of the atmosphere is 14·7 lbs. per sq. in. Determine the corresponding equivalents in feet, also in inches of water. 1 cu. ft. of water weighs 62·5 lbs.

12. 1 cu. in. of steel weighs 0·28 lb.; determine the weight of 1 cu. ft.

13. Two scale drawings are made of two cubes; the scale of the first is 1″=2 ft., and of the second 1″=5 ft. Both cubes have an edge of 3·75 ins. on the drawings. Find the ratio of (1) their volumes, (2) their surfaces.

14. A cubical tank open at the top is $\frac{2}{3}$ full of water. What percentage of its inner surface is wet?

15. The surface of a cube is 0·125 its volume; find its edge.

16. The volume of a cube is $\frac{1}{60}$ its surface; determine the edge.

17. A cube of steel 5·95″ edge is hammered when hot into a rectangular prism whose cross-section is 7·3″ × 2·1″. Determine the length of the prism.

18. The ratio of the dimensions of a rectangular prism is 1 : 3 : 4, and its volume is 768 cu. cms. Determine its dimensions. Make a scale drawing.

19. A square prism is 3″ × 3″ × 9·5″; determine its volume and surface, and draw a plan and elevation to scale.

20. A square pillar is 19·27 cms. high and has a volume of 876 cu. cms. Determine the side of the square.

21. A rectangular prism is 2·5″ × 3·3″ × 9·7″. Determine the length of a diagonal. Draw a plan and elevation to scale and show diagonal.

22. A rectangular prism is 3″ × 2″ × 7″; determine the radius of its circumscribing sphere. Draw a plan and elevation to scale.

23. The surface of a rectangular prism is 562 sq. ins., and the ratio of the dimensions is 2 : 3 : 5. Determine the length, breadth and thickness.

24. A piece of steel of $\frac{3}{4}$″ square section is chosen to make a lathe tool. Determine the weight, if its length is 7·25″. 1 cu. in. of steel = 0·28 lb.

25. If the weight in (24) was 3 lbs., what length was chosen?

26. For a certain contract the following bars are necessary: $\frac{1}{2}$″ sq. steel bar 10 ft., $\frac{1}{4}$″ sq. steel bar 14 ft., $\frac{3}{8}$″ × 1″ rectangular bar 9 ft., $1\frac{1}{2}$″ × $2\frac{3}{4}$″ rectangular bar 35 ft. Determine the total weight. 1 cu. in. of steel = 0·28 lb.

27. A cube of steel 5·73″ edge is penetrated axially by a square hole 3·59″ side. Determine the volume of the metal.

28. 1 cu. ft. of water weighs 62·5 lbs. Find the volume of 37·5 lbs. in cubic inches.

29. A bar of steel $2\frac{1}{2}$″ square and 2′ long is rolled into a square bar 12′ long. Find the dimensions of the bar after rolling.

30. An electric accumulator 12″ × 15″ × 12″ high contains sulphuric acid to a depth of $8\frac{1}{2}$″. If the specific gravity of the acid is 1·205, find its weight. 1 cu. ft. of water = 62·5 lbs.

31. A vessel displaces 25,600 tons of fresh water. How many cu. ft. of salt water will it displace? 1 cu. ft. of salt water = 64 lbs.

32. The extension of a helical spring is proportional to the load. A bar of metal 2″ × $1\frac{1}{4}$″ × 3″ produces an extension of 0·42″, 12 lbs. produces an extension of 2″. Find the weight of 1 cu. in. of the metal.

33. 1 gram of water has a volume of 1 cu. cm.,
1 lb. ,, weighs 453·6 grams.
1 cu. ft. ,, ,, 62·5 lbs.

Find the number of cu. cms. in 1 cu. ft. of water.

34. A and B are two equidimensional tanks, bases 20′ × 25′, heights 12′. A is full of fresh water (1 cu. ft. = 62·5 lbs.). B has the same

weight of salt water as A has fresh water (1 cu. ft. salt water = 64 lbs.). Find the level of the water in B.

35. The normal pressure of the atmosphere is equivalent to a column of mercury 76 cms. high. Find the pressure in lbs. per sq. inch due to the atmosphere. 1 cu. cm. of mercury = 13·6 grams, 453·6 grams = 1 lb. (Take a column of mercury 1 sq. in. in section 76 cms. high, and find its weight.)

36. A wedge whose cross-section is an isosceles triangle, base 1·43″, altitude 3·85″, is 5·2″ long. Find its volume.

37. A steel knife edge whose cross-section is an equilateral triangle $\frac{3}{8}$″ side is $1\frac{3}{8}$″ long. Find its volume and weight. 1 cu. in. steel = 0·28 lb.

38. A steel plate—rhombus with sides $8\frac{1}{2}$″ long and base angle 60°—is $\frac{1}{4}$″ thick. Find its weight if 1 cu. in. steel plate = 0·283 lb.

39. A steel plate—rhombus with sides 5″ long and base angle 80°—weighs 4·9 lbs. Find its thickness. 1 cu. in. steel plate = 0·283 lb.

40. Find the weight of 10′ of hexagonal steel rod, the side of the hexagon being $\frac{3}{4}$″. 1 cu. in. of steel = 0·28 lb.

41. A cotter 8″ long is $\frac{3}{4}$″ wide at the thin end. It has a taper of 1 in 16. Find its volume and weight if it is $\frac{5}{8}$″ thick throughout its length. 1 cu. in. = 0·28 lb. (See taper in Appendix.)

CHAPTER VI.

THE CYLINDER.

If a rectangle ABCD is revolved about its base DC, the surface generated is a cylindrical one, and the solid is termed a cylinder (Fig. 97).

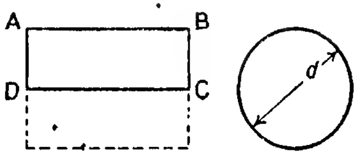


Fig. 97.

The line DC is the axis of the cylinder, and is at right angles to each plane end. Moreover, the axis is an imaginary line joining the centre of each end.

Since the sections of the cylinder by all planes at 90° to the axis are equal circles, it follows that its volume will be the area of one circle × length of cylinder. Such a solid is termed a right circular cylinder.

$$\begin{aligned} \text{Volume} &= \mathrm{A} \times h \\ &= \pi r^2 h \\ &= \frac{\pi d^2 h}{4} \\ &= \underline{0{\cdot}7854 d^2 h.} \end{aligned}$$

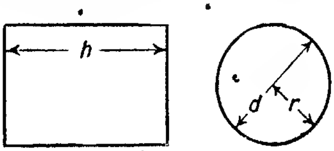


Fig. 98.

If a piece of paper was wrapped round the cylinder and then unfolded (this would be a development of its curved surface) its shape would be rectangular (Fig. 99).

The depth of the rectangle would be the circumference of a circle, *i.e.* $2\pi r$ or πd, and its length h.

Hence the area of its curved surface

$$\begin{aligned} &= 2\pi r h \\ &= \underline{\pi d h.} \end{aligned}$$

The area of each plane end $= \pi r^2$

$$= \frac{\pi d^2}{4};$$

and there being two ends, the total surface

$=$ curved surface + two plane ends

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h+r)$

$= \pi d(h + d/2).$

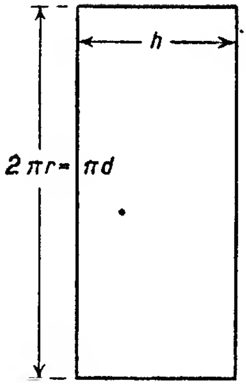


Fig. 99.

The general definition of a cylinder is as follows:

When one extremity of a line moves round any plane closed curve in such a way that it is parallel to itself in every position, a cylinder is generated. Moreover, a cylindrical surface is a surface of revolution.

Let Fig. 100 represent any plane closed curve, *i.e.* a plane curve enclosing an area; then, if from every part of its circumference lines are drawn perpendicular to its plane, the lines lie on a cylindrical surface. This method of forming the cylinder is precisely the same as that stated above. Should the lines be drawn obliquely (all having the same inclination) to the plane, the solid will be a cylinder. (a) and (b) show the plan and elevation of a right cylinder. It is termed thus, because it is formed by lines perpendicular to the base. If the lines are not perpendicular, it would be an oblique cylinder (Fig. 101).

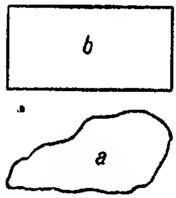
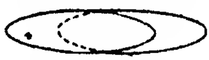


Fig. 100.

l = slant height, h = altitude or perpendicular height.



Generating curve.

Fig. 101.

The cross-section of a cylinder by all planes parallel to the base is identical with the curve of generation, *i.e.* the original curve. Since all cylindrical surfaces are formed by the revolution of straight lines, it follows that they may be developed, *i.e.* laid out flat without creasing. Consequently the area of the curved surface is the perimeter of generating curve × altitude (a parallelogram* is the plane figure representing the development).

$$\text{Area} = \underline{\text{per. of curve} \times \text{altitude} = Ph.}$$
$$\text{Vol.} = \underline{\text{area of base} \times \text{altitude} = Ah.}$$

An oblique cylinder could be obtained from a right cylinder by assuming the right cylinder to consist of a very large number of very thin discs. By causing each disc to slide sideways an amount proportional to its distance from the base, an oblique cylinder would result. The above is equivalent to subjecting the cylinder to shear.

Elliptical Cylinder. An elliptical cylinder is one whose cross-section parallel to the base is an ellipse (Fig. 102).

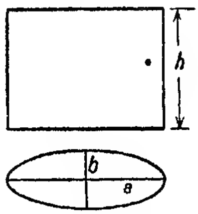


FIG. 102.

$$\text{Curved surface} = \underline{\pi(a+b)h} \text{ approximately.}$$
$$\text{Plane surfaces} = \underline{2\pi ab.}$$
$$\text{Volume} = \underline{\pi abh.}$$

Hollow Cylinder.

Volume = volume of external cylinder − volume of internal cylinder.

* If cylinder is right, the figure is a rectangle.

In the case of a right circular cylinder this would be

$$\pi(r_1^2 - r_2^2)h = \frac{\pi}{4}(d_1^2 - d_2^2)h$$

$= \pi d_m th$ (see p. 54)

$=$ mean circumference $\times$ thickness $\times$ height

$=$ mean circumference $\times$ cross-sectional area.

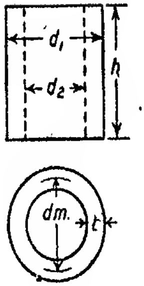


FIG. 103.

EXAMPLES.

1. The diameter of a right circular cylinder is 53·9 cms. Find its volume and total surface if its height is 119 cms.

$$\text{Vol.} = \frac{\pi d^2 h}{4}$$

$$= \frac{\pi \times 53{\cdot}9^2 \times \cancel{119}^{\,29{\cdot}75}}{\cancel{4}}$$

$= 271{,}600$ cubic cms.

Curved surface + plane surface

$$= \pi dh + \frac{2\pi d^2}{4}$$

$$= \pi d(h + d/2)$$

$$= \pi \times 53{\cdot}9\left(119 + \frac{53{\cdot}9}{2}\right)$$

$$= \pi \times 53{\cdot}9(119 + 26{\cdot}95)$$

$$= \pi \times 53{\cdot}9 \times 146,$$

i.e. total surface $= 21{,}710$ sq. cms.

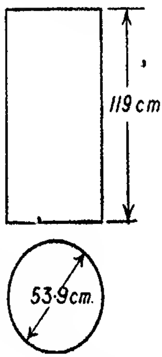


FIG. 104.

2. The volume of a cylinder is 123·9 cu. ins., and the ratio of the height to the radius 1 : 0·72.
Determine the magnitude of each.

$$\text{Vol.} = \pi r^2 h;$$

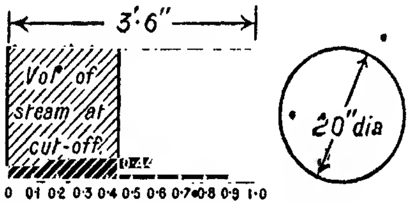
$$\therefore \ \pi r^2 h = 123{\cdot}9. \quad \text{.........................(1)}$$

Now
$$\frac{r}{h}=\frac{0\cdot72}{1};$$
$$\therefore\ r=0\cdot72h.$$

Substituting in (1), we have
$$\pi\times0\cdot72^2h^3=123\cdot9;$$
$$\therefore\ h^3=\frac{123\cdot9}{\pi\times0\cdot72^2};$$
$$\therefore\ h=\sqrt[3]{\frac{123\cdot9}{\pi\times0\cdot72^2}}$$
$$=\underline{4\cdot237''.}$$

But
$$r=0\cdot72h$$
$$=0\cdot72\times4\cdot237,$$
$$\text{i.e.}\quad r=\underline{3\cdot051\text{ ins.}}$$

3. An engine cylinder is 20″ dia., 3′ 6″ stroke, and cuts off steam at 0·44 stroke, the steam being supplied at 200 lbs. per sq. in. Find the cylinder feed per stroke in lbs., also the consumption per hour at 75 r p m. 1 lb. steam at 200 lbs. per sq. in. = 2·26 cu. ft. (The stroke is the effective length of the cylinder.)



FRACTIONS OF STROKE. Cut Off = 0·44.

FIG. 105.

Cut off is the point during the stroke of the piston at which the supply of steam from the boiler is stopped. Since no more steam is supplied, the steam in the cylinder expands and does work as the piston moves forward.

Volume of cylinder at 0·44 stroke

$$= 0{\cdot}44\frac{\pi d^2 h}{4}$$

$$= \frac{\overset{0\cdot11}{\cancel{0\cdot44}}}{\cancel{4}}\pi \times \left(\frac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{12}}}\right)^2 \times 3{\cdot}5$$

$$= \frac{0{\cdot}11\pi \times 25 \times \overset{0\cdot3889}{\cancel{3\cdot5}}}{\cancel{9}}$$

$$= 3{\cdot}359 \text{ cubic feet.}$$

Cylinder feed per stroke = volume at cut off × wt. of 1 cu. ft.

$$= \frac{3{\cdot}359}{2{\cdot}26}$$

$$= 1{\cdot}487 \text{ lbs.}$$

This is of course the feed when conditions are ideal. The actual amount of steam supplied would be much greater on account of cylinder condensation and leakage. An estimate of the steam used can be made by comparison with the performance of similar engines.

Consumption per hour = strokes per hour × feed per stroke

$$= 2 \times \text{r.p.m.} \times 60 \times 1{\cdot}487$$

$$= 2 \times 75 \times 60 \times 1{\cdot}487$$

$$= 9000 \times 1{\cdot}487$$

$$= 13{,}383 \text{ lbs. of steam, say } 13{,}400.$$

This again is the consumption under ideal conditions.

4. A boiler shell is 3′ 6″ dia., 16′ long and $\frac{1}{2}$″ thick. Find its weight if 1 sq. ft. of $\frac{1}{8}$″ steel plate = 5·1 lbs. (Do not include the ends.)

Area of shell = πdh, where d is the diameter and h the overall length.

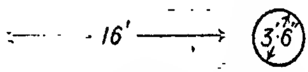


FIG. 106.

Strictly speaking, each section or ring of the shell ought to be calculated separately, the different diameters and the

overlaps being taken into account, together with the rivet heads, cover plates, etc. The method shown gives a fair approximation.

$$A = \pi dh$$
$$= \pi \times 3{\cdot}5 \times 16.$$

Wt. of shell = superficial area × wt. of 1 sq. ft. of $\frac{1}{2}''$ plate

$$= \pi \times 3{\cdot}5 \times 16 \times 5{\cdot}1 \times 4 \quad (4 \times \tfrac{1}{8}'' = \tfrac{1}{2}'')$$
$$= \pi \times 56 \times 20{\cdot}4$$
$$= \underline{3589 \text{ lbs. or } 1{\cdot}603 \text{ tons, say } 3600 \text{ lbs. or } 1{\cdot}61 \text{ tons.}}$$

In this case the external dia. has been taken. The error in so doing is of no practical importance.

5. A flywheel rim is 9′ external dia., 8′ internal dia. and 1′ 2″ wide. Determine its weight and the total weight of the wheel. 1 cu. in. cast iron = 0·26 lb.

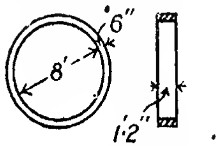


Fig. 107.

Volume of rim = mean circumference × area (see p. 109)

$$= \pi \times \frac{(8+9)}{\cancel{2}} \times \overset{6}{\cancel{12}} \times 6 \times 14 \quad d_m = \left(\frac{8+9}{2}\right) \times 12$$
$$= \pi \times 17 \times 6 \times 84$$
$$= \pi \times 102 \times 84$$
$$= \underline{26{,}930 \text{ cubic ins.}}$$

Weight of rim = vol. in cu. ins. × wt. of 1 cu. in.

$$= 26{,}930 \times 0{\cdot}26.$$
$$= \underline{7000 \text{ lbs.}}$$
$$= \underline{3{\cdot}125 \text{ tons.}}$$

Weight of boss and arms = $\frac{1}{2}$ wt. of rim (about)

$$= \frac{3{\cdot}125}{2}$$
$$= 1{\cdot}563 \text{ tons or } 3500 \text{ lbs.}$$

$$\therefore \text{ total wt.} = 7000 + 3500$$
$$= \underline{10{,}500 \text{ lbs.}}$$
$$= 3{\cdot}125 + 1{\cdot}563$$
$$= \underline{4{\cdot}688 \text{ tons, say } 4{\cdot}7.}$$

When calculating the steadying effect of a flywheel, it is usual to consider the rim alone, this being adequate for all practical purposes. In this respect the wt. of the rim alone is needed.

6. A cylinder 7″ dia. fits in a cubical box. Calculate the percentage void.

$$\text{Void} = \text{vol. of cube} - \text{vol. of cylinder}$$
$$= 7^3 - \frac{\pi \times 7^2 \times 7}{4}$$
$$= 7^3\left\{1 - \frac{\pi}{4}\right\}$$
$$= 7^3\{1 - 0{\cdot}7854\}$$
$$= \underline{7^3 \times 0{\cdot}2146}$$

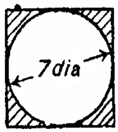


FIG. 108.

$$\% \text{ void} = \frac{\text{vol. of void}}{\text{vol. of cube}} \times 100$$
$$= \frac{\cancel{7^3} \times 0{\cdot}2146 \times 100}{\cancel{7^3}}$$
$$= \underline{21{\cdot}46.}$$

Notice the method of calculation adopted in this case. It simplifies matters considerably.

7. The diagram shows the elevation of a hexagonal bolt. Determine the length of the shaft, so that it has the same weight as the head. (Height of head $= \frac{3}{4}''$.) Neglect the chamfer.

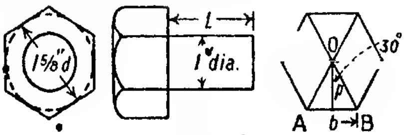


FIG. 109.

If the weights are equal, so also are the volumes.

Hence vol. of head = vol. of shaft.

$\therefore$ area of hexagon $\times \frac{3}{4}$ = area of shaft $\times\, l$. ....(1)

The diagonals of a regular hexagon divide it into six equal equilateral triangles.

$$\text{Area of AOB} = p \times b$$
$$= \frac{1{\cdot}625}{2} \times b. \quad (1\tfrac{5}{8}'' = 1{\cdot}625'')$$

Now
$$\frac{b}{p} = \tan 30^\circ;$$
$$\therefore \ b = p \times \tan 30^\circ$$
$$= \frac{1{\cdot}625}{2} \times 0{\cdot}5774.$$
$$\therefore \ \text{area of AOB} = \frac{1{\cdot}625}{2} \times \frac{1{\cdot}625}{2} \times 0{\cdot}5774$$
$$= \frac{(1{\cdot}625)^2 \times 0{\cdot}5774}{4};$$
$$\therefore \ \text{area of hexagon} = \frac{6 \times (1{\cdot}625)^2 \times 0{\cdot}5774}{4}.$$

Substituting in (1), we obtain

$$\frac{\overset{3}{\cancel{6}} \times (1{\cdot}625)^2 \times 0{\cdot}5774}{\cancel{4}} \times \frac{\overset{1{\cdot}5}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \frac{\pi \times 1^2}{\cancel{4}} \cdot l;$$
$$\therefore \ l = \frac{(1{\cdot}625)^2 \times 0{\cdot}5774 \times 4{\cdot}5}{\pi}$$
$$= \underline{2{\cdot}183''}.$$

From the numerical value of the area of the hexagon stated above, we can deduce that the area of a regular hexagon circumscribed about a circle of radius a is

$$6a^2 \times 0{\cdot}5774 = \frac{6a^2}{\sqrt{3}}, \quad 0{\cdot}5774 = \tan 30^\circ = \frac{1}{\sqrt{3}}.$$

8. Determine the discharge in cubic feet and lbs. per hour from a thin lipped circular orifice 3″ dia. when the velocity of exit is 23 feet per sec. Coefficient of discharge $= 0{\cdot}62$.

Suppose the water issued from a cylindrical pipe 3 ins. dia. In one second every particle of water in the pipe, between the end and 23 ft. from the end, would be discharged.

$$\text{Its volume} = \frac{\pi \times 3^2}{4 \times 144} \times 23 \text{ cubic feet,}$$

i.e. the discharge per sec. = area of orifice in sq. ft. × velocity in ft. per sec.

This, however, is the theoretical discharge. The actual discharge is only 0·62 of this, owing to friction and contraction of the jet at the orifice.

$$\therefore \text{ discharge per sec.} = \frac{\pi \times \cancel{3^2}}{\cancel{4} \times \cancel{144}_{16}} \times 23 \times \cancelto{0\cdot155}{0\cdot62};$$

225

$$\therefore \text{ discharge per hour} = \frac{\pi \times 23 \times 0\cdot155 \times \cancelto{900}{3600}}{\cancelto{4}{16}}$$

$$= \underline{2520 \text{ cu. ft.}}$$

$$\left.\begin{array}{r}\text{Discharge in lbs.}\\ \text{per hour}\end{array}\right\} = \text{cu. ft. per hour} \times \text{wt. of 1 cu. ft.}$$

$$= 2520 \times 62\cdot5$$

$$= \underline{157{,}500.}$$

9. The diagram shows a coil of leather belting 0·22″ thick. Determine its length.

Suppose the belting to be unwound and laid out flat, then the volume of the prism so obtained is equal to the volume of the hollow cylinder* shown in the diagram (Fig. 110).

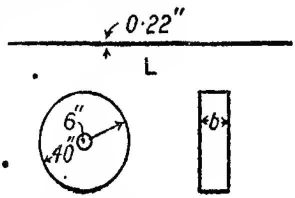


Fig. 110.

Let $\quad L = \text{length of belting.}$

Then $\quad L \times 0\cdot22\cancel{b} = \pi(r_1^2 - r_2^2)\cancel{b};$

$$\therefore L = \frac{\pi}{0\cdot22}(r_1 + r_2)(r_1 - r_2)$$

$$= \frac{\pi(20+3)(20-3)}{0\cdot22}$$

$$= \frac{\cancelto{100}{22} \times \cancelto{3\cdot286}{23} \times 17}{\cancel{7} \times \cancel{0\cdot22}}$$

$$= 328\cdot6 \times 17$$

$$= \underline{5584'' \text{ or } 465\cdot33',}$$

$$\underline{\text{say } 5580'' \text{ or } 465'.}$$

* The coil is a hollow cylinder approximately.

Examples to be Worked Out.

1. Calculate the volumes and curved surfaces of the following cylinders:*

	Dia.	Height.		Rad.	Height.
(*a*)	72·5″	89·5″	(*f*)	2·31″	0·279″
(*b*)	81·25 cms.	32·72 cms.	(*g*)	0·276 cms.	27·16 cms.
(*c*)	9·23′	12·81′	(*h*)	1·25′	18·32′
(*d*)	1·235″	1·57″	(*i*)	3·93″	12·76″
(*e*)	0·065″	0·024″	(*j*)	0·127″	3·95″

Draw a plan and elevation of each to scale.

2. The area of the curved surface of a cylinder is 18·59 sq. cms. and its length 0·35 metre. Find its radius.

3. The volume of a cylinder is 923·6 cu. ins. and its length 25″. Determine the radius.

4. The volume of a cylinder is 236·2 cu. ins. and the ratio of the radius to the height 1 : 1·85. Determine the radius and height.

5. The curved surface of a cylinder is 823 sq. ft. Determine its dimensions, if the ratio of the height to the radius is 3 : 1·76.

6. An engine cylinder is 24″ diameter. Find the area of the cylinder walls and piston exposed to steam (1) at 0·4 stroke, (2) at 1·0 stroke, the crank radius being 2′. (The crank radius is half the stroke. The stroke is the effective length of the cylinder.)

7. If the above engine cuts off steam at 0·35 stroke, and the steam up to cut off is dry and is supplied at 195 lbs. per sq. in.; find the cylinder feed per stroke in lbs. Also find the steam consumption per hour. R.p.m. = 80. 1 lb. of steam at 195 lbs. per sq. in. occupies a volume 2·31 cu. ft.

8. A cylindrical water main is 3′ internal dia. and is horizontal. It is full of water to a point 2′ above the bottom. What percentage of its inner surface is wet (see p. 85)?

9. A Lancashire boiler shell is 30′ long ½″ thick and 8′ diameter. Find its weight if 1 sq. ft. of ½″ steel plate weighs 5·1 lbs.

10. The commutator of a dynamo is 26″ dia. and 18″ long. Find the radiating surface in square decimetres, *i.e.* the curved surface. 1″ = 0·254 dm.

11. The length of the core of a dynamo is 25″, and it must have a radiating surface of at least 6000 sq. in. Determine its minimum diameter. (The core is cylindrical, the radiating surface being the curved surface.)

12. A disc flywheel is 3′ 9″ dia. and 6″ thick. Determine its weight if one cubic inch of cast iron = 0·26 lb.

13. Find the weight of the flywheel shown. 1 cu. in. C.I. = 0·26 lb.

14. A flywheel rim is 10′ external and 8′ 9″ internal diameter and 15″ wide. Determine its weight and the total weight of the wheel. 1 cu. in. C.I. = 0·26 lb.

* Cylinders are right circular unless otherwise stated.

15. A water main is 2′ 6″ internal diameter and 1″ thick. Find the weight of 25′. 1 cu. in C. I. = 0·26 lb.

16. A cylinder, 9″ diameter and 9″ long, fits in a cubical box. Determine the percentage void.

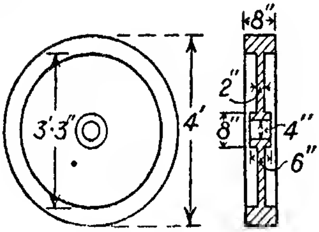
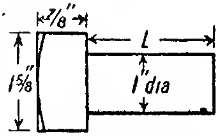


Fig for Ex. 13

17. Four equal cylinders, each 9·75″ long, fit in a cubical box 9·75″ edge. Determine the percentage void.

18. The diagram shows the elevation of a square-headed bolt Determine the length of the shaft to have twice the weight of the head.



19. A roller for a bowling green is 4′ diameter and 8′ ft. wide. It makes 10 turns in going across the green. What area of grass is rolled?

20. Determine the discharge in cubic feet per hour from a thin-lipped circular orifice $5\frac{1}{2}''$ dia. The coefficient of discharge is 0·62 and the velocity of the outflowing water 17·5 ft. per sec

21. An elliptical funnel has a major axis of 30′ and a minor axis of 20′. Find the discharge of smoke in cu ft. per min. at a velocity of 10 ft. per sec.

22. Find the conditions that a right elliptical cylinder the ratio of whose axes is 2·5 : 1 shall have the same volume as a circular cylinder of half the height.

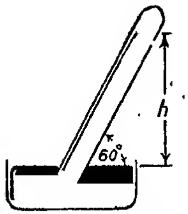
23. A roll of paper is 3′ external diameter, and the diameter of the pole on which it is wound is 8″. If the paper is 0·005″ thick, determine its length.

24. A leather belt 100 yds long and 0·22″ thick is coiled on a rod 10″ diameter. Determine the external diameter of the coil.

25. In the design of compound engines, *i.e.* engines in which the steam does work by expansion in more than one cylinder, a rule often adopted is, volume of H.P.C. : volume of L.P.C. = 1 : 3. If the diameter of the H.P.C. is 15″ and the stroke of both pistons 3′ 6″, *i.e.* the length of each cylinder is 3′ 6″, calculate (*a*) volume of H.P.C., (*b*) volume of L.P.C., (*c*) dia. of L.P.C. (H.P.C. means high-pressure cylinder; L.P.C. means low-pressure cylinder.)

26. On the assumptions that in (25) the cylinder feed per stroke is 0·65 lb. steam, rev. per min. 110, determine the steam consumption per hour. Allowing that 19·5 lbs. steam per hour develop 1 horse-power for 1 hour, calculate the power of the engine.

27. If a barometer tube is inclined to the horizon, the height h of the mercury in the tube above the level of the reservoir is constant. Suppose the tube to have a bore of 5 mms. and the barometric height to be 760 mms.; find the volume of mercury above the reservoir level.



28. A marine engine has three cylinders whose diameters are in the ratio 3 : 5 : 8. Each cylinder has an effective length of 36″, i.e. the stroke is 36″ and the diameter of the smallest cylinder is 12″. Find the other two diameters and the ratio of the cylinder volumes.

29. A locomotive has 225 cylindrical brass fire tubes, each 1⅞″ external diameter and 11′ 3″ long. Find the heating surface. In addition there is the heating surface of the firebox, which is 159·1 sq. ft. What is the total heating surface? If the grate area is 20·5 sq. ft., find the heating surface per sq. ft. of grate area.

30. Find the volume and weight of the given cast-iron plate. 1 cu. in. of cast iron = 0·26 lb. Make a scale drawing.

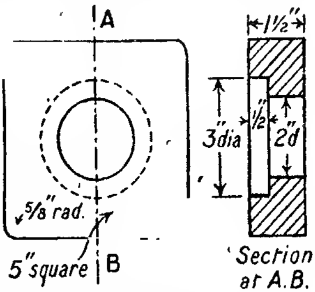
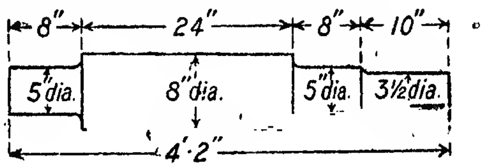


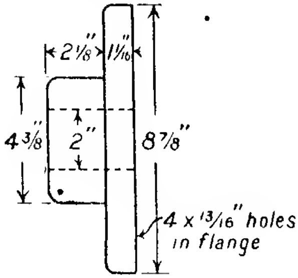
Fig. for Ex. 30.

31. An elliptical plate weighs 20 lbs. The major and minor axes of the ellipse are 14″ and 10″. Find its thickness if 1 cu. in. steel plate = 0·283 lb.

32. Find the weight of the shafting shown. 1 cu. in. steel = 0·28 lb. Neglect fillets. Draw to scale and add an end elevation.



33. Find the weight of coupling shown. Neglect rounding of corners. Draw the given view to scale and add an end elevation.

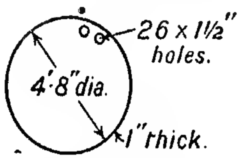


34. A cylinder 5·9" dia. is cut by an oblique plane at 50°, the intersection of the plane and the axis being 7·27" from the base; determine the volume of the solid so obtained.

35. A cube 3·87 cms. edge is inscribed in a right circular cylinder. Find the dimensions of the cylinder.

36. A cube is inscribed in a right circular cylinder 0·348 cm. dia. Find its edge.

37. A cast-iron plate is shown in sketch. Calculate its weight. 1 cu. in. = 0·26 lb.



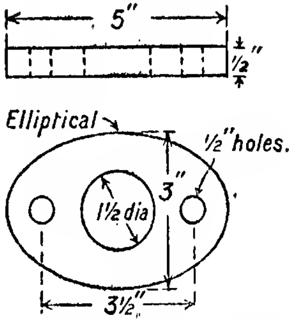
38. A cylindrical graduated glass jar 1" dia. contains 500 c.cms. of water. Find the distance between consecutive graduations of 1 c.c. State the result in inches.

39. Three equal right circular cylinders fit in a hollow equilateral triangular prism 5" edge. Find the percentage void. (The cylinders are the same length as the prism.)

40. The weight of 1 yard of standard rail is 95 lbs. Find the cross-sectional area in sq. ins. 1 cu. in. = 0·28 lb.

41. What would be the diameter of a burette, the distance between the c.c. marks being 1 cm.? What practical advantage and disadvantage has a burette, (1) large bore, (2) small bore?

42. Find the weight of the given portion of a cast-iron gland. 1 cu. in. $= 0{\cdot}26$ lb.



43. A shaft weighing 808 lbs. has a uniform cross-section and is 25 ft. long. Find its diameter if 1 cu. in. $= 0{\cdot}28$ lb.

44. A building has to be supplied with 5700 cu. ft. of air per minute for ventilation purposes. An electric motor making 655 r.p.m. is available for driving the centrifugal fan. The quantity discharged per revolution per sq. in. of discharge pipe area is 4·5 cu. ins. Find the area of the pipe in sq. ft. and the velocity of flow.

45. Two cylinders, each 10" dia. and 15" long, rest on the horizontal plane with their axes parallel and 15" apart. They are cut by another horizontal plane in such a way that the area of the portion of the plane intercepted by each cylinder is equal to the area of the portion of the plane between the cylinders. Find the two positions of the plane to fulfil the above condition. Draw a plan and elevation to scale

CHAPTER VII.

THE CONE.

If a right-angled triangle ABC is revolved about its base AC, the surface generated is a conical one, and the solid is termed a cone (Fig. 111).

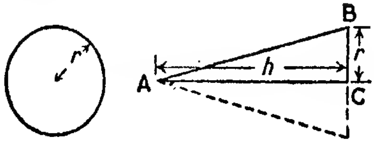


Fig. 111

AC, the axis of the cone, is an imaginary line joining the vertex to the centre of the base. In the present instance, the axis is at right angles to the base, and the sections by all planes at 90° to the axis are circles, the radii of which depend on the positions of the planes relatively to the vertex. Such a solid is known as a right circular cone.

Volume $= \frac{1}{3}Ah$, where A = area of base,

$= \underline{\frac{1}{3}\pi r^2 h}$, *i.e.* $\frac{1}{3}$ vol. of a cylinder having the same altitude and radius of base as the cone.

Since every point on the circumference of the base is equidistant from the apex, the development of a conical surface will be a sector of a circle whose radius is the slant height of the cone (Fig. 112).

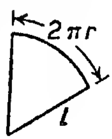


Fig. 112.

Area of curved surface $=$ arc $\times \frac{\text{rad.}}{2}$ (see p. 78)

$= 2\pi r \cdot \frac{l}{2},$

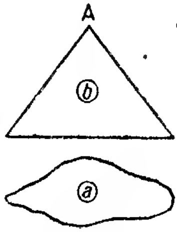
$= \underline{\pi r l}.$

Area of plane end or base $= \underline{\pi r^2}$;

$$\therefore \text{ total surface} = \pi r^2 + \pi r l$$
$$= \underline{\pi r(r+l)}.$$

A cone, like a cylinder, can have any closed curve for its cross-section (Fig. 113). The general definition of a conical surface is as follows: When one extremity of a straight line passes through a fixed point, and the other extremity moves so that the line is always on the circumference of any plane closed curve, a cone is generated. Moreover, a conical surface is a surface of revolution.

Let A be the apex or vertex of the cone. Then, if from A lines are drawn to every part of the plane closed curve shown in the plan, the lines lie on a conical surface. This method gives precisely the same result as that stated above.



a = plan.
b = elevation.
FIG. 113.

When the base is a figure symmetrical about two perpendicular axes, *e.g.* an ellipse (an ellipse is symmetrical about its major and minor axes), and the apex is immediately above the centre of the base, the axis is at 90° to the base, and on account of this the solid is a right cone.

Should the axis not be at 90° to the base, the solid is an oblique cone.

The sections of a cone by planes parallel to the base or plane of generation, are similar figures.

The volume of any cone $= \underline{\frac{1}{3} \times \text{area of base} \times \text{altitude}}$.

No general formula can be given for the curved surface of a cone.

Elliptical Cone.

$$\text{Volume} = \tfrac{1}{3}A \times h$$
$$= \underline{\tfrac{1}{3}\pi abh}.$$

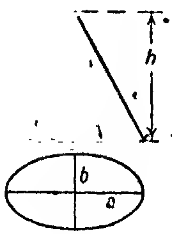
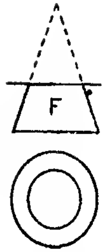


FIG. 114.

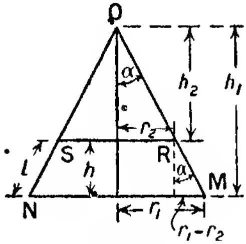
Truncated Cone, Frustum. When any cone is cut by planes dividing it into two or more portions it is said to be truncated.* The portions which do not contain the vertex, *i.e.* the portions between successive planes, are called frusta (Fig. 115).



F = frustum.

Fig. 115

To find formulae giving the volume and curved surface of a circular conical frustum.



α is the semi-vertical angle of the cone.

Fig. 116.

Given r_1, r_2 and h. Complete the cone, and dimension it as shown (Fig. 116).

$$\text{Volume of frustum} = \tfrac{1}{3}\pi(h_1 r_1^2 - h_2 r_2^2).$$

We have
$$\left.\begin{aligned} \frac{r_2}{h_2} &= \tan\alpha, \\ \frac{r_1}{h_1} &= \tan\alpha, \end{aligned}\right\} \qquad \begin{aligned} \therefore\ \frac{r_2}{h_2} &= \frac{r_1}{h_1}\,; \\ \therefore\ h_2 &= \frac{r_2}{r_1} \times h_1. \end{aligned}$$

Substituting in the above formula, we obtain

$$V = \tfrac{1}{3}\pi\left(h_1 r_1^2 - \frac{r_2}{r_1} \times h_1 \times r_2^2\right)$$
$$= \tfrac{1}{3}\pi\frac{h_1}{r_1}(r_1^3 - r_2^3).$$

* In the cases treated herein, the planes are at 90° to the axis of the cone.

Now, $\frac{r_1 - r_2}{h} = \tan\alpha$;

$$\therefore \frac{r_1 - r_2}{h} = \frac{r_1}{h_1}, \quad \text{since} \quad \tan\alpha = \frac{r_1}{h_1};$$

$$\therefore \frac{h}{r_1 - r_2} = \frac{h_1}{r_1}, \quad \text{and} \quad V = \tfrac{1}{3}\pi \times \frac{h}{r_1 - r_2}(r_1^3 - r_2^3)$$

$$= \underline{\tfrac{1}{3}\pi h(r_1^2 + r_1 r_2 + r_2^2)} \quad \ldots\ldots\ldots\ldots(1)$$

$$= \tfrac{1}{3}h(\pi r_1^2 + \sqrt{\pi r_1^2}\sqrt{\pi r_2^2} + \pi r_2^2)$$

$$= \tfrac{1}{3}h(A_1 + \sqrt{A_1 A_2} + A_2),$$

A_1 being the area of the lower end and A_2 of the upper. The last formula is general,* and holds no matter what shape the ends may have, provided they are parallel.

If we substitute $r_2 = 0$ in (1), $V = \frac{1}{3}\pi h r_1^2$, which is the volume of a cone untruncated. This condition could be obtained by assuming the cutting plane to move upwards until it reached the vertex. In this position $r_2 = 0$.

If $r_1 = r_2$, $V = \frac{1}{3}\pi h(r_1^2 + r_1^2 + r_1^2) = \pi h r_1^2$, which is the volume of a cylinder. In this case the apex of the cone would be situated at an infinite distance, and the angle of the cone would be infinitely small.

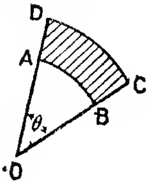


Fig. 117.

Curved surface = ABCD (Fig. 117)

$$= \tfrac{1}{2}\theta(OC^2 - OB^2)$$

$$= \frac{1}{2}\frac{\text{arc } DC}{CO}(OC^2 - OB^2) \quad \text{(see p. 68)}$$

$$= \frac{1}{2}\frac{\text{arc } DC}{OM}(OM^2 - OR^2) \quad \text{(see Fig. 116).}$$

Now $$\frac{r_1}{OM} = \sin\alpha, \quad \therefore\ OM = \frac{r_1}{\sin\alpha}.$$

$$\frac{r_2}{OR} = \sin\alpha, \quad \therefore\ OR = \frac{r_2}{\sin\alpha}.$$

$$\text{Hence C.S.} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}\pi \cancel{r_1}}{\frac{\cancel{r_1}}{\sin\alpha}}\left(\frac{r_1^2}{\sin^2\alpha} - \frac{r_2^2}{\sin^2\alpha}\right)$$

* The above is not a proof of the general case.

$$= \pi\,\cancel{\sin\alpha}\left(\frac{r_1^2 - r_2^2}{\sin^{\cancel{2}}\alpha}\right)$$

$$= \frac{\pi}{\sin\alpha}(r_1^2 - r_2^2)$$

$$= \frac{\pi}{\frac{r_1 - r_2}{l}}(r_1^2 - r_2^2)$$

$$= \pi l\,\frac{(r_1^2 - r_2^2)}{r_1 - r_2}$$

$$= \underline{\pi l(r_1 + r_2).}$$

This reduces to $\pi l r_1$ when $r_2 = 0$, *i.e.* the C.S. of a cone untruncated.

EXAMPLES.

1. The altitude of a cone is 32·1 cms. and the radius of its base 9 cms. Find its volume, curved surface, plane surface and total surface. Find its weight if it is made of cast iron. 1 cu. in. $= 0{\cdot}26$ lb.

$$\text{Volume} = \tfrac{1}{3}\pi r^2 h$$

$$= \tfrac{1}{3}\pi \times 9^2 \times 32{\cdot}1$$

$$= \tfrac{1}{\cancel{3}}\pi \times \overset{27}{\cancel{81}} \times 32{\cdot}1$$

$$= \underline{2724 \text{ cubic cms.}}$$

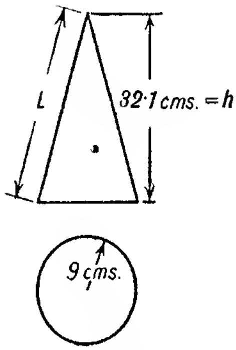


FIG. 118.

A cylinder having the same radius of base and altitude as this cone would have a volume of $3 \times 2724 = \underline{8172 \text{ cu. cms.}}$

Curved surface $= \pi r l$.

Now $l^2 = r^2 + h^2$, since the triangle is rt. angled. (See Fig. 119.)

$$\therefore\ l = \sqrt{r^2 + h^2}$$
$$= \sqrt{9^2 + 32{\cdot}1^2}$$
$$= \sqrt{81 + 1030}$$
$$= \sqrt{1111}$$
$$= \underline{33{\cdot}34 \text{ cms.}}$$

$$\therefore\ \text{C.S} = \pi \times 9 \times 33{\cdot}34$$
$$= \underline{942{\cdot}6 \text{ sq. cms.}}$$

$$\text{Plane surface} = \pi r^2$$
$$= \pi \times 81$$
$$= \underline{254{\cdot}5 \text{ sq. cms.}}$$

$$\text{Hence total surface} = \text{C.S.} + \text{P.S.}$$
$$= 942{\cdot}6 + 254{\cdot}5$$
$$= \underline{1197{\cdot}1 \text{ sq. cms., say } 1197.}$$

$$\text{Weight} = \text{volume} \times \text{weight of unit volume}$$
$$= \frac{2724}{2{\cdot}54^3} \times 0{\cdot}26 \qquad \left(\begin{array}{r} 1'' = 2{\cdot}54 \text{ cms.} \\ \therefore\ 1 \text{ cu. in.} = 2{\cdot}54^3 \text{ cu. cms.}\end{array}\right)$$
$$= \underline{43{\cdot}23 \text{ lbs.}}$$

2. The curved surface of a cone is 18·37 sq. ins. and the ratio of the altitude to the radius 4 : 3. Determine its dimensions.

$$\text{C.S.} = \pi r l.$$

Now $\quad l = \sqrt{r^2 + h^2}.$

But $\quad \frac{h}{r} = \frac{4}{3}\,;$

$$\therefore\ h = \tfrac{4}{3} r,$$

and $\quad l = \sqrt{r^2 + (\tfrac{4}{3} r)^2}$

$$= \sqrt{r^2 + \tfrac{16}{9} r^2}$$
$$= r\sqrt{\tfrac{25}{9}},$$
$$\text{i.e. } l = \underline{\tfrac{5}{3} r.}$$

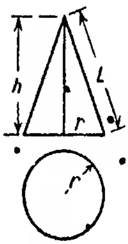


FIG. 119.

Substituting in above equation, we have

$$\text{C.S.} = \pi r \times \tfrac{5}{3} r$$
$$= \tfrac{5}{3}\pi r^2;$$
$$\therefore \ \tfrac{5}{3}\pi r^2 = 28 \cdot 37;$$
$$\therefore \ r^2 = \frac{3 \times \overset{3\cdot674}{\cancel{18\cdot37}}}{\cancel{5}\pi};$$
$$\therefore \ r = \sqrt{\frac{11\cdot022}{\pi}}$$
$$= \underline{1\cdot873''}.$$
$$h = \tfrac{4}{3} r$$
$$= \tfrac{4}{3} \times \overset{0\cdot624}{\cancel{1\cdot873}}$$
$$= \underline{2\cdot496'', \text{ say } 2\cdot5''}.$$

3. A cone 5″ rad. 12″ high is divided into three equal volumes by two planes at 90° to the axis. Determine the position of each plane, and find the areas intercepted by each plane.

Volume of cone $= \frac{1}{3}\pi r_1^2 h$;

$\therefore$ vol. of each portion

$$= \tfrac{1}{9}\pi r_1^2 h$$
$$= \frac{\pi}{\underset{3}{\cancel{9}}} \times 5^2 \times \overset{4}{\cancel{12}}$$
$$= \underline{\frac{100\pi}{3}}.$$

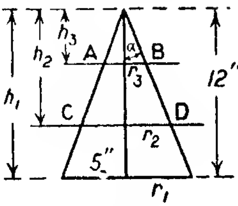


Fig. 120.

Vol. of upper part of height $h_3 = V_3 = \frac{1}{3}\pi r_3^2 h_3$.

Now $\qquad \dfrac{r_3}{h_3} = \tan \alpha = \dfrac{5}{12}. \quad \therefore \ r_3 = \dfrac{5}{12} h_3.$

$\therefore$ by substitution, $V_3 = \dfrac{1}{3}\pi \left(\dfrac{5}{12}\right)^2 h_3^2 \times h_3$

$$= \frac{\pi}{3} \times \frac{25}{144} \times h_3^3.$$

But $\qquad$ each volume $= \dfrac{100\pi}{3},$

whence
$$\frac{\cancel{\pi}}{\cancel{3}}\times\frac{\cancel{25}}{144}h_3^3=\frac{\overset{4}{\cancel{100}}\cancel{\pi}}{\cancel{3}};$$
$$\therefore\ h_3^3=4\times144$$
$$=576;$$
$$\therefore\ h_3=\sqrt[3]{576}$$
$$=\underline{8\cdot32''.}$$

Volume of cone of height $h_2=\mathsf{V}_2=\frac{1}{3}\pi r_2^2h_2$.

Now
$$\frac{r_2}{h_2}=\tan\alpha=\frac{5}{12},\quad\therefore\ r_2=\frac{5}{12}h_2;$$
$$\therefore\ \text{by substitution }\mathsf{V}_2=\frac{1}{3}\pi\left(\frac{5}{12}\right)^2h_2^2\times h_2$$
$$=\frac{\pi}{3}\times\frac{25}{144}h_2^3.$$

But
$$\mathsf{V}_2=2\times\frac{100\pi}{3};$$
$$\therefore\ \frac{\cancel{\pi}}{\cancel{3}}\times\frac{\cancel{25}}{144}h_2^3=\frac{2\times\overset{4}{\cancel{100}}\cancel{\pi}}{\cancel{3}};$$
$$\therefore\ h_2^3=8\times144$$
$$=1152;$$
$$\therefore\ h_2=\sqrt[3]{1152}$$
$$=\underline{10\cdot48''.}$$

Area intercepted by plane $\mathsf{AB}=\pi r_3^2$
$$=\pi\times\left(\frac{5}{12}h_3\right)^2$$
$$=\frac{\pi\times25}{144}\times8\cdot32^2$$
$$=\frac{\pi\times100\times\cancel{8\cdot32^2}}{\cancel{144}\times\cancel{4}}=\frac{314\cdot2}{8\cdot32}$$
$$8\cdot32$$
$$=\underline{37\cdot77\text{ sq. ins.}}$$

Area intercepted by plane CD $= \pi r_2^2$

$$= \pi\left(\frac{5}{12}h_2\right)^2$$

$$= \frac{\pi \times 25}{144} \times 10\cdot48^2$$

$$= \frac{200\pi}{144 \times 8} \times 10\cdot48^2$$

$$= \frac{200\pi \times \cancel{10\cdot48^2}}{\cancel{1152}}$$
$$10\cdot48$$

$$= \frac{628\cdot32}{10\cdot48}$$

$$= \underline{59\cdot94 \text{ sq. ins.}}$$

Notice the artifices adopted in calculating the areas intercepted by the planes.

4. The clevation of a cone is an isosceles triangle $6'' \times 6'' \times 4''$. Determine its altitude, and draw a development of the curved surface, after calculating the necessary data.

$$h^2 + 2^2 = 6^2;$$
$$\therefore\ h^2 = 6^2 - 2^2$$
$$= 36 - 4$$
$$= 32;$$
$$\therefore\ h = \sqrt{32}$$
$$= \underline{5\cdot657''}.$$

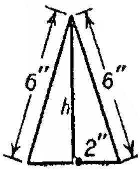


FIG. 121.

$$\theta = \frac{\text{arc}}{\text{radius}}.$$
$$= \frac{2\pi r}{l}$$
$$= \frac{2\pi \times \cancel{2}}{\cancel{6}}$$
$$3$$
$$= \underline{\frac{2}{3}\pi \text{ radians}}$$
$$= \frac{2}{\cancel{3}} \times \overset{60}{\cancel{180}}$$
$$= \underline{120 \text{ degrees.}}$$

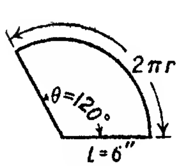


FIG. 122.

5. The velocity of water at the entrance to a conical nozzle is 15 feet per sec.; what must be the diameter at exit, and the length of the nozzle, if the taper is 1 in 10 and the velocity of discharge 40 ft. per sec.? Dia. at entrance 6″. Friction and contraction of the jet at exit to be neglected.

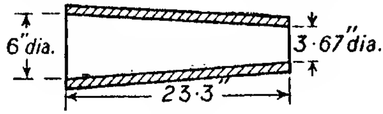


FIG. 123.

Quantity flowing through first section (at entrance) per sec. = Quantity flowing through second section (at exit) per sec.

$$\therefore \frac{\pi d_1^2 v_1}{4} = \frac{\pi d_2^2 v_2}{4};$$

$$\therefore d_2^2 = d_1^2 \times \frac{v_1}{v_2};$$

$$\therefore d_2 = d_1\sqrt{\frac{v_1}{v_2}}$$
$$= 6\sqrt{\frac{15}{40}}$$
$$= 6\sqrt{0\cdot375}$$
$$= 6 \times 0\cdot6124$$
$$= \underline{3\cdot67''}.$$

The taper is 1 in 10, *i.e.* the diameter decreases 1 in. for every 10 in. length of nozzle. (See Appendix.)

Now decrease of diameter $= 6 - 3\cdot67$
$= \underline{2\cdot33''}.$

Hence length $= 2\cdot33 \times 10$
$= \underline{23\cdot3''}.$

6 A right elliptical cone has a volume of 97·5 cu. ins., and the ratio of the major to the minor axis 3·5 : 2. The inclina-

tion of the lines joining the extremities of the major axis to the vertex is 69°. Determine the dimensions of the cone.

$$\text{Volume} = \tfrac{1}{3}\pi abh.$$

Now $h = a\tan 69°$

$= a \times 2{\cdot}6051,$

and $\dfrac{b}{a} = \dfrac{2}{3{\cdot}5}$;

$\therefore\ b = \dfrac{2a}{3{\cdot}5} = \dfrac{a}{1{\cdot}75}.$

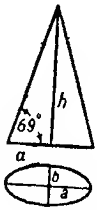


Fig. 124.

$\therefore$ by substitution,

$$V = \frac{1}{3}\pi a \times \frac{a}{1{\cdot}75} \times a \times 2{\cdot}605$$

$$= \frac{2{\cdot}605\pi a^3}{5{\cdot}25};$$

$$\therefore\ \frac{2{\cdot}605\pi a^3}{5{\cdot}25} = 97{\cdot}5;$$

$$\therefore\ a^3 = \frac{97{\cdot}5 \times 5{\cdot}25}{2{\cdot}605 \times \pi};$$

$$\therefore\ a = \sqrt[3]{\frac{97{\cdot}5 \times 5{\cdot}25}{2{\cdot}605 a\pi}}$$

$$= \underline{3{\cdot}97''}.$$

Whence $2a$, the major axis $= 2 \times 3{\cdot}97 = \underline{7{\cdot}94''}$.

$h = 3{\cdot}97 \times 2{\cdot}605$ (by substituting in above formula)

$= \underline{10{\cdot}34''}.$

$b = \dfrac{a}{1{\cdot}75}$

$= \dfrac{3{\cdot}97}{1{\cdot}75}$

$= \underline{2{\cdot}27''}.$

Whence $2b$, the minor axis $= 2 \times 2{\cdot}27 = \underline{4{\cdot}54''}$.

7. A right elliptical cone, major axis 12·57″, minor axis 9·85″, altitude 15·72″, is cut by a plane parallel to the base

and 8·75″ from the vertex. Find the area intercepted by the plane.

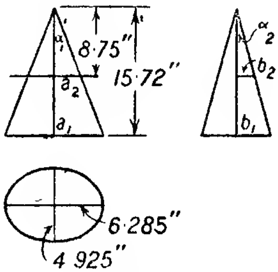


FIG. 125.

$$\left.\begin{aligned} \frac{a_1}{15\cdot72} &= \tan\alpha_1 = \frac{a_2}{8\cdot75}, \\ \frac{b_1}{15\cdot72} &= \tan\alpha_2 = \frac{b_2}{8\cdot75}, \end{aligned}\right\} \quad \begin{aligned} \therefore\ \frac{a_1}{15\cdot72} &= \frac{a_2}{8\cdot75}; \\ \therefore\ \frac{b_1}{15\cdot72} &= \frac{b_2}{8\cdot75}. \end{aligned}$$

Whence $$\frac{a_1b_1}{15\cdot72^2} = \frac{a_2b_2}{8\cdot75^2};$$

$$\therefore\ a_2b_2 = \left(\frac{8\cdot75}{15\cdot72}\right)^2 a_1b_1.$$

$\therefore\ \pi a_2b_2$, the area of the intercepted portion,

$$= \pi a_1b_1\left(\frac{8\cdot75}{15\cdot72}\right)^2$$

$$= \pi \times 6\cdot285 \times 4\cdot925 \times \left(\frac{8\cdot75}{15\cdot72}\right)^2$$

$$= \underline{30\cdot12 \text{ sq. ins.}}$$

Since the area $= \pi a_1b_1\left(\frac{8\cdot75}{15\cdot72}\right)^2$, we see that it is proportional to the square of the distance from the vertex. This is true for cones no matter what shapes their bases may have.

8. Two bevel wheels, having 40 and 60 teeth $\frac{3}{4}''$ pitch, gear together. Find the diameters and angles of the pitch cones.

The diagram (Fig. 126) shows the bevel wheels connecting two shafts at right angles. AB is the diameter of wheel (1) (when considering bevel wheels the mean diameter is taken);

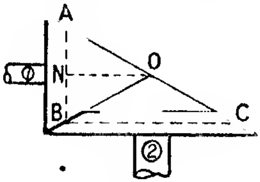


Fig. 126.

BC is the diameter of wheel (2). AOC is a straight line, $A\hat{B}C = 90°$. ABO is one imaginary pitch cone and BCO the other. The cones must have a common vertex for the wheels to gear together correctly. The pitch of the teeth is $\frac{3}{4}''$ on the circles, whose diameters are AB and BC. On circles nearer O it is less than $\frac{3}{4}''$, and on circles more remote from O it is greater than $\frac{3}{4}''$.

As in the case of spur wheels, we have

Circumference of pitch circle = pitch × number of teeth,

i.e. $\pi d = pn$; (see Chapter III. p. 59)

$$\therefore d = \frac{pn}{\pi}$$

$$= \frac{\frac{3}{4} \times \overset{10}{\cancel{40}}}{\pi}$$

$$= \frac{30}{\pi},$$

i.e. dia. = 9·548″ for wheel (1).

$$d = \frac{pn}{\pi}$$

$$= \frac{\frac{3}{4} \times \overset{15}{\cancel{60}}}{\pi}$$

$$= \frac{45}{\pi},$$

i.e. dia. = 14·322″ for wheel (2).

$$\frac{AN}{ON} = \tan A\hat{O}N;$$

$$\therefore \tan A\hat{O}N = \frac{\frac{AB}{2}}{\frac{BC}{2}}$$

$$= \frac{AB}{BC}$$

$$= \tfrac{40}{60}$$

$$= \tfrac{2}{3} = 0{\cdot}6667 \text{ correct to 4 places;}$$

$$\textit{i.e.} \quad A\hat{O}N = \tan^{-1} 0{\cdot}6667$$

$$= 33^\circ\ 41';$$

$$\therefore A\hat{O}B = 2A\hat{O}N = \underline{67^\circ\ 22'}.$$

(See interpolation in Appendix.)

$$A\hat{O}B + B\hat{O}C = 180;$$

$$\therefore B\hat{O}C = 180^\circ - A\hat{O}B$$

$$= 180^\circ - 67^\circ\ 22'$$

$$= \underline{112^\circ\ 38'}.$$

9. The height of a conical frustum is 5″ and the radii of the plane ends 7″ and 5″. Find its volume and curved surface.

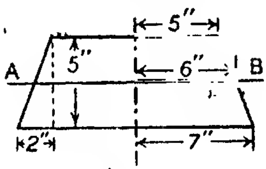


Fig. 127.

$$\text{Volume} = \frac{\pi h}{3}\{r_1^2 + r_1 r_2 + r_2^2\}$$

$$= \frac{\pi \times 5}{3}\{7^2 + 7 \times 5 + 5^2\}$$

$$= \frac{\pi \times 5}{3}\{49 + 35 + 25\}$$

$$=\frac{\pi\times 5}{\cancel{3}}\times\frac{36\cdot33}{\cancel{100}}$$
$$=\pi\times 181\cdot65$$
$$=\underline{570\cdot9 \text{ cu. ins.}}$$

The volume may be found approximately as follows

$$\text{Vol.}=\text{area of middle section}\times\text{ height}$$
$$=\pi\left(\frac{r_1+r_2}{2}\right)^2\times h$$
$$=\pi r_m^2\times h,\ r_m \text{ being the mean radius,}$$
$$=\pi\left(\frac{7+5}{2}\right)^2\times 5$$
$$=\pi\times 6^2\times 5$$
$$=\pi\times 180$$
$$=\underline{565\cdot6 \text{ cu. ins.}}$$

The error is $570\cdot9-565\cdot6=5\cdot3$;

$$\%\text{ error}=\frac{5\cdot3\times100}{570\cdot9}$$
$$=\underline{0\cdot93.}$$

This error would, as a general rule, be negligible in practice. The percentage error depends solely on the ratio of r_1 to r_2.

When $r_1=1\cdot1r_2$ the error is $0\cdot07\%$,
" $r_1=1\cdot3r_2$ " $0\cdot53\%$,
" $r_1=1\cdot5r_2$ " $1\cdot32\%$.

The error is always negative, *i.e.* the result is too small. In the present case $r_1=1\cdot4r_2$.

$$\text{Curved surface}=\pi l(r_1+r_2).$$
$$l=\sqrt{5^2+2^2}=\sqrt{25+4}=\sqrt{29}$$
$$=\underline{5\cdot385''.}$$

Hence
$$\text{C.S.}=\pi\times5\cdot385(7+5)$$
$$=\pi\times5\cdot385\times12$$
$$=\pi\times64\cdot62$$
$$=\underline{203 \text{ sq. ins.}}$$

If $\frac{r_1}{r_2}$ is almost unity, the curved surface is $\pi h(r_1+r_2)$ approximately,

$$\doteq 2\pi\left(\frac{r_1+r_2}{2}\right)h,$$

because $h=l$ almost, *i.e.* the

C.S. = mean circumference or circumference of middle section $\Bigg\}$ × altitude (approximately).

Example 20 at the end of this chapter is a case in point.

Examples to be Worked Out.

1. Find the volumes, curved surfaces and plane surfaces of cones* having the following dimensions:

(*a*) $r=3{\cdot}25''$, $h=6''$. (*b*) $r=92{\cdot}5''$, $h=85''$.
(*c*) $r=0{\cdot}125'$, $h=2{\cdot}25'$. (*d*) dia. $=6{\cdot}25$ yds., $h=9{\cdot}3$ yds.

Draw a plan and elevation of each to scale.

2. A right elliptical cone has a major axis 15″, minor axis 12″ and altitude 10″. Determine its volume. Draw a plan and elevation to scale.

3. The volume of a right circular cone is 857·3 cu. cms., and the ratio of the height to the radius 2·75 : 1. Determine its dimensions, and draw a plan and elevation to scale.

4. The curved surface of a right circular cone is 36·2 sq. ins. and the ratio of the altitude to the radius 3 : 2. Determine its dimensions, and draw a plan and elevation to scale.

5. A right elliptical cone has a volume of 81·3 cu. mms. and the ratio of the major to the minor axis 2·5 : 1. The inclination of the lines joining the extremities of the major axis to the apex is 65°. Determine its dimensions.

6. A cylinder of lead 5″ dia. and 12″ long is recast into a cone whose radius is 10·3″. Find its altitude.

7. A cone has a vertical angle of 78° and an altitude of 13·8′. Find the dimensions of the development of its curved surface, and draw it to scale.

8. A sector of a circle 15′ rad., angle 135°, is the development of a conical tent. Find the height of the centre pole and the semi-vertical angle of the tent.

9. A right circular cone 7″ rad. and 15″ high is cut by a plane, at right angles to its axis, into two portions of equal volume. Determine the position of the plane.

10. If the cone in (9) was truncated by two parallel planes, at right angles to its axis, giving 3 equal volumes, find the position of both planes.

11. If the cone in (9) was divided by a plane into two portions whose curved surfaces were equal, find the position of the plane.

* Cones are right circular unless otherwise stated.

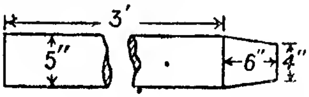
12. If the cone in (9) was divided into two portions such that the area intercepted by the plane was $\frac{1}{2}$ the area of the base, find the position of the plane.

13. Show that the area intercepted by any plane at right angles to the axis of a right circular cone is proportional to the square of its distance from the vertex.

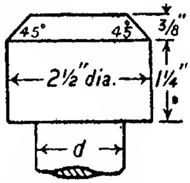
14. The elevation of a right circular cone is an equilateral triangle 8″ side. Find its altitude and curved surface. Draw a development after calculating the necessary data.

15. The velocity of water at the entrance to a conical nozzle is 12 ft. per sec.; what must be the dia. at exit and the length of the nozzle, if the velocity of the issuing jet is 30 ft. per sec. and the taper 1 in 10? Neglect friction. Dia. at entrance 5″.

16. Find the wt. of the steel piston rod in sketch. 1 cu. in. of steel = 0·28 lb.



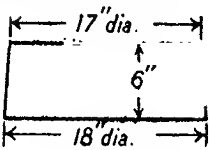
17. The diagram shows the elevation of a cheese head (with a bevel) for a bolt. Find its wt. 1 cu. in. of steel = 0·28 lb.



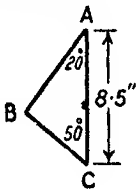
18. A conical frustum with parallel ends has radii 3·2″ and 6·9″, and a height of 5·75″. Find its volume and curved surface. Draw to scale a plan and elevation.

19. A conical frustum has a volume of 91·8 cu. cms. and the ratio of the radii 2·69 : 4·95. Determine each radius.

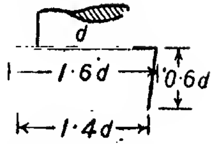
20. The diagram shows the elevation of a conical friction clutch. Find the contact area in sq. ins., *i.e.* the area of the curved surface.



21. If the triangle ABC revolves about AC, determine by aid of an accurate scale drawing the volume and curved surface so generated.



22. The sketch shows the usual proportions of a pan-headed rivet. Find the weight of a rivet head for a $\frac{5}{8}''$ dia. rivet. 1 cu. in. $=0{\cdot}28$ lb. Hence deduce the weight of a rivet head $1\frac{1}{4}''$ dia. Neglect rounding of corners.



23. A cube $3''$ edge is inscribed in a right circular cone, semi-vertical angle 30°. Find the height of the cone.

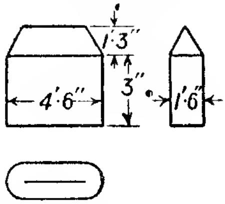
24. A hole $4''$ dia. is bored axially through a cone $10''$ high and $6''$ rad. of base. Find the volume and curved surface of the remaining solid. Draw a plan and elevation to scale.

25. A pail for holding water has a base $15''$ dia. and a top $18''$ dia. The height of the pail is $20''$. Determine its volume and the area of metal used for its construction.

26. If the water level in (25) was $4''$ below the top, find the amount it contained.

27. A right circular cone is $12''$ high and $5''$ radius, and is cut by horizontal planes at distances of $3''$, $6''$, $9''$ from the vertex. Show that the ratio of the volumes included between each plane and the vertex is $1:8:27$.

28. Find the weight of $\frac{3}{16}''$ plate necessary to construct the following. Draw it to scale. 1 sq. ft. $\frac{1}{8}''$ steel plate $=5{\cdot}1$ lbs.



29. Find the weights of cones (*a*) and (*b*) in (1) if 1 cubic inch $=0{\cdot}26$ lb.

30. Find the weight of the cone in (2) if 1 cubic inch $=0{\cdot}32$ lb.

31. Two bevel wheels, having 50 and 70 teeth $\frac{1}{2}''$ pitch, gear together. Find the diameters and angles of the pitch cones. (Shafts at 90°.)

32. A conical boiler flue is $5'\ 9''$ long. The ends are 4 ft. and $2'\ 9''$ dia. respectively. Find the wt. of $\frac{1}{2}''$ steel plate necessary for its construction, no allowance being made for overlap. Draw a development to scale. 1 sq. ft. $\frac{1}{8}''$ steel plate $=5{\cdot}1$ lbs.

33. Find what volume of air is contained by a street lamp in the form of a truncated cone. Rad. of large end $12''$, rad. of small end $8''$, height $24''$.

CHAPTER VIII.

THE SPHERE.

IF a semicircle ABC is revolved about its diameter AB, the surface generated is a spherical one, and the solid is termed a sphere (Fig. 128).

Every point on the surface is equidistant from the centre O. All lines drawn from the centre to the surface are radii.

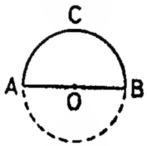


FIG. 128.

Any section of a sphere by a plane is a circle. When the plane passes through the centre, the circle so obtained is a great circle; otherwise it is a small circle. The surface of a sphere is not developable, *i.e.* it cannot be laid out flat uncreased.

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{\pi d^3}{6}$$

$$= 0{\cdot}5236 d^3.$$

Superficial area of a sphere $= 4\pi r^2 =$ four times the area of a great circle

$$= \pi d^2.$$

If a sphere is hollow, the external and internal radii being r_1 and r_2, the

$$\text{Volume} = \tfrac{4}{3}\pi r_1^3 - \tfrac{4}{3}\pi r_2^3$$

$$= \tfrac{4}{3}\pi(r_1^3 - r_2^3)$$

$$= \frac{\pi}{6}(d_1^3 - d_2^3) = 0{\cdot}5236(d_1^3 - d_2^3),$$

d_1 and d_2 being the diameters external and internal.

Provided the thickness is small compared with the internal radius, no serious error is involved in taking the volume as the product of the mean surface and the thickness.

The % error depends solely on the ratio of r_1 to r_2.

When $r_1 = 1{\cdot}1r_2$ the error is $0{\cdot}07$ %,
„ $r_1 = 1{\cdot}3r_2$ „ $0{\cdot}53$ %,
„ $r_1 = 1{\cdot}5r_2$ „ $1{\cdot}32$ %.

The error is always negative, *i.e.* the volume is too small.

In cases where the radii differ by a very small amount, the external or internal surface may be taken, *e.g.* a boiler with hemispherical ends.

$$\text{Vol.} = 4\pi r_m^2 t, \text{ where } r_m = \frac{r_1 + r_2}{2} = \text{mean rad.}$$

$$= \underline{\pi d_m^2 t}, \ d_m \text{ being the mean diameter.}$$

$$\left(d_m = \frac{d_1 + d_2}{2}.\right)$$

Spherical Segment. When a plane divides a sphere into two portions each is called a segment (Fig. 129).

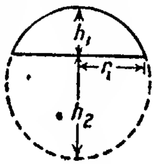


Fig. 129.

$$\text{Volume of upper segment} = \underline{\frac{\pi h_1}{2} r_1^2 + \frac{\pi h_1^3}{6}}.$$

Curved surface of upper segment

$$= \underline{2\pi r h_1 = \pi d h_1},$$

where r is the radius of the *sphere* and d the diameter.

$$\text{Volume of lower segment} = \underline{\frac{\pi h_2}{2} r_1^2 + \frac{\pi h_2^3}{6}}.$$

$$\text{Curved surface of lower segment} = \underline{2\pi r h_2}$$

$$= \underline{\pi d h_2}.$$

$$\text{Plane surface of each segment} = \underline{\pi r_1^2}.$$

Determination of r.

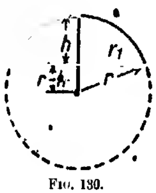


Fig. 130.

$$r^2 = (r-h)^2 + r_1^2;$$

$$\therefore \ \cancel{r^2} = \cancel{r^2} - 2rh + h^2 + r_1^2;$$

$$\therefore \ 2rh = r_1^2 + h^2;$$

$$\therefore \ r = \frac{r_1^2 + h^2}{2h}.$$

Spherical Zone. When a sphere is divided into three portions by two parallel planes, the portion between the planes is termed a zone. The remaining portions are segments (Fig. 131).

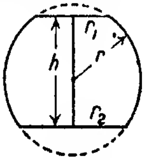


FIG. 131.

$$\text{Volume of zone} = \frac{\pi h}{2}(r_1^2 + r_2^2) + \frac{\pi h^3}{6}.$$

If we imagine the lower plane to move downward parallel to itself till it is tangent to the sphere, $r_2 = 0$, and we get two segments.

The above formula becomes $V = \frac{\pi h}{2} r_1^2 + \frac{\pi h^3}{6}$, which is the formula for the volume of a segment.

Suppose this plane to remain where it is, and let the upper plane move upwards till it is tangent to the sphere.

$r_1 = 0$, $r_2 = 0$ and $V = \frac{\pi h^3}{6} = \frac{\pi d^3}{6}$, the volume of the sphere.

Curved surface of zone $= 2\pi rh = \pi dh$, r being the radius of the sphere and d the diameter.

In the case just mentioned, $h = d = 2r$.

Whence $\text{C.S.} = 2\pi r \times 2r$

$= 4\pi r^2$, the formula for the surface of the sphere.

Plane surface of zone $= \pi r_1^2 + \pi r_2^2$

$$= \pi(r_1^2 + r_2^2) = \frac{\pi}{4}(d_1^2 + d_2^2)$$

$$= 0{\cdot}7854\,(d_1^2 + d_2^2).$$

Determination of r.

$$\left.\begin{aligned} r^2 &= r_1^2 + h_1^2, \\ r^2 &= r_2^2 + h_2^2. \end{aligned}\right\}$$

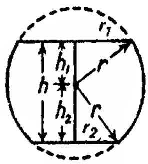


FIG. 132.

Subtracting, we have $r_1^2 + h_1^2 - r_2^2 - h_2^2 = 0$.

But $h = h_1 + h_2$;

$$\therefore\ h_2 = h - h_1;$$

$$\therefore\ r_1^2 + h_1^2 - r_2^2 - (h - h_1)^2 = 0;$$

$$\therefore\ r_1^2 + \cancel{h_1^2} - r_2^2 - h^2 + 2hh_1 - \cancel{h_1^2} = 0;$$

$$\therefore\ 2hh_1 = r_2^2 - r_1^2 + h^2;$$

$$\therefore\ h_1 = \frac{r_2^2 - r_1^2 + h^2}{2h}.$$

The value of r may be found by substituting for h_1 in the equation $r^2 = r_1^2 + h_1^2$.

EXAMPLES.

1. A copper sphere has a radius of 2·35″; determine its volume, surface and wt. 1 cu. in. = 0·32 lb.

$$\text{Volume} = \tfrac{4}{3}\pi r^3$$
$$= \tfrac{4}{3}\pi \times 2{\cdot}35^3$$
$$= \underline{54{\cdot}39 \text{ cu. ins.}}$$
$$\text{Or vol.} = 0{\cdot}5236 d^3$$
$$= 0{\cdot}5236 \times 4{\cdot}7^3$$
$$= \underline{54{\cdot}39 \text{ cu. ins.}}$$

FIG. 133.

Notice that the second method is less laborious.

$$\text{Surface} = 4\pi r^2$$
$$= 4\pi \times 2{\cdot}35^2$$
$$= \underline{69{\cdot}42 \text{ sq. ins.}}$$
$$\text{Or surface} = \pi d^2$$
$$= \pi \times 4{\cdot}7^2$$
$$= \underline{69{\cdot}42 \text{ sq. ins.}}$$

The second method is again shorter.

$$\text{Weight} = \text{volume} \times 0{\cdot}32$$
$$= 54{\cdot}39 \times 0{\cdot}32$$
$$= \underline{17{\cdot}4 \text{ lbs.}}$$

2. A sphere has a volume of 325·1 cu. cms.; determine its radius.

$$V = 0{\cdot}5236 d^3;$$
$$\therefore\ d^3 = \frac{V}{0{\cdot}5236};$$
$$\therefore\ d = \sqrt[3]{\frac{325{\cdot}1}{0{\cdot}5236}}$$
$$= 8{\cdot}54''$$
$$\therefore\ r = \frac{8{\cdot}54}{2}$$
$$= \underline{4{\cdot}27''.}$$

3. Find the volume of a hollow sphere 1·9″ internal and 3·4″ external radii.

$$\begin{aligned}\text{Vol.} &= 0{\cdot}5236(d_1{}^3 - d_2{}^3)\\ &= 0{\cdot}5236(6{\cdot}8^3 - 3{\cdot}8^3)\\ &= 0{\cdot}5236(314{\cdot}4 - 54{\cdot}87)\\ &= 0{\cdot}5236 \times 259{\cdot}53\\ &= \underline{135{\cdot}9 \text{ cu. ins., say } 136 \text{ cu. ins.}}\end{aligned}$$

The above formula must be applied in a case like this, because the thickness is not small compared with the internal radius.

4. A hollow spherical shell is 19″ ext and 18″ int. dia. Find its volume and weight. 1 cu. in. = 0·28 lb.

$$\begin{aligned}\text{Vol.} &= \pi d_m^2 t\\ &= \pi\left(\frac{19+18}{2}\right)^2 \times 0{\cdot}5\\ &= \pi \times 18{\cdot}5^2 \times 0{\cdot}5\\ &= \underline{537{\cdot}8 \text{ cu. ins., say } 538 \text{ cu. ins.}}\end{aligned}$$

$$\begin{aligned}\text{Weight} &= \text{vol.} \times 0{\cdot}28\\ &= 537{\cdot}8 \times 0{\cdot}28\\ &= \underline{150{\cdot}6 \text{ lbs., say } 151 \text{ lbs.}}\end{aligned}$$

To show the accuracy of the above method of estimating the volume, we will calculate the exact volume and compare the two results.

$$\left.\begin{array}{r}\text{Exact vol.,}\\ \text{i.e. assuming } \pi = 3{\cdot}1416,\end{array}\right\} \begin{aligned} &= 0{\cdot}5236(d_1{}^3 - d_2{}^3)\\ &= 0{\cdot}5236(19^3 - 18^3)\\ &= 0{\cdot}5236(6859 - 5832)\\ &= 0{\cdot}5236(1027)\\ &= \underline{537{\cdot}6 \text{ cu. ins.}}\end{aligned}$$

Hence the error is $537{\cdot}8 - 537{\cdot}6 = 0{\cdot}2$.

$$\text{Percentage error} = \frac{0{\cdot}2 \times 100}{537{\cdot}6} = \frac{20}{537{\cdot}6} = \text{about } \frac{1}{27},$$

which is negligible.

5. A sphere 10″ dia. rests on the H.P. and is cut by two H.P.'s at distances of 4″ and 8″ from the point of contact with

the first plane. Calculate the volume and curved surface of each of the three portions.

C.S. of spherical segment (1)

$$= \pi d h_1$$
$$= \pi \times 10 \times 4$$
$$= 40\pi$$
$$= 40 \times 3{\cdot}1416$$
$$= \underline{125{\cdot}7 \text{ sq. ins.}}$$

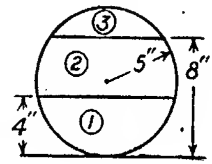


FIG. 134.

$$\text{C.S. of } (2) = \pi d h_2$$
$$= \pi \times 10 \times 4$$
$$= \underline{125{\cdot}7 \text{ sq. ins.}}$$

$$\text{C.S. of } (3) = \pi d h_3$$
$$= \pi \times 10 \times 2$$
$$= \underline{62{\cdot}83 \text{ sq. ins.}},$$

i.e. half of each of the two former.

$$r_1^2 + 1^2 = 5^2;$$
$$\therefore\ r_1^2 = 25 - 1;$$
$$\therefore\ r_1^2 = \underline{24}.$$

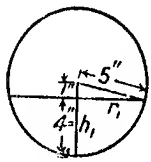


FIG. 135.

$$\text{Vol. of } (1) = \frac{\pi h_1}{2} \times r_1^2 + \frac{\pi h_1^3}{6}$$
$$= \pi\left(\frac{h_1 r_1^2}{2} + \frac{h_1^3}{6}\right)$$
$$= \pi\left(\frac{\overset{2}{\cancel{4}} \times 24}{\cancel{2}} + \frac{\overset{32}{\cancel{64}}}{\underset{3}{\cancel{6}}}\right)$$
$$= \pi(48 + 10{\cdot}67)$$
$$= 58{\cdot}67\pi$$
$$= \underline{184{\cdot}4 \text{ cu. ins.}}$$

$$r_2^2 + 3^2 = 5^2;$$
$$\therefore\ r_2^2 = 25 - 9$$
$$= \underline{16}.$$

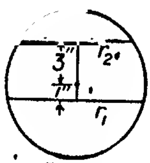


FIG. 136.

$$\text{Vol. of (2)} = \frac{\pi h_2}{2}(r_1^2 + r_2^2) + \frac{\pi h_2^3}{6}$$

$$= \frac{\pi h_2}{2}\left(r_1^2 + r_2^2 + \frac{h_2^2}{3}\right)$$

$$= \frac{\pi \times \overset{2}{\cancel{4}}}{\cancel{2}}\left(24 + 16 + \frac{\overset{5\cdot33}{\cancel{16}}}{\cancel{3}}\right)$$

$$= 2\pi(45\cdot33)$$

$$= 90\cdot66\pi$$

$$= \underline{284\cdot8 \text{ cu. ins.}}$$

$$\text{Vol. of (3)} = \frac{\pi h_3}{2}(r_2^2) + \frac{\pi h_3^3}{6}$$

$$= \frac{\pi h_3}{2}\left(r_2^2 + \frac{h_3^2}{3}\right)$$

$$= \frac{\pi \times \cancel{2}}{\cancel{2}}\left(16 + \frac{4}{3}\right)$$

$$= \pi \times 17\cdot33$$

$$= \underline{54\cdot44 \text{ cu. ins.}}$$

6. A hemispherical bowl 8″ dia. with half its surface wet rests on the H.P. and is tilted until the water is just on the point of overflowing. Find (1) the distance of water level from the centre; (2) the angle α; (3) the distance of A from H.P.

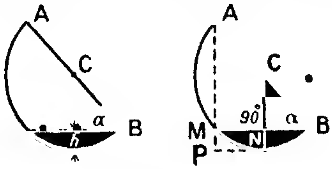


FIG. 137.

$$\text{Surface} = \frac{\pi d^2}{2}$$

$$= \frac{\pi \times 8^2}{2}$$

$$= \underline{32\pi \text{ sq. ins.}}$$

$$\therefore \text{ surface wet} = 16\pi \text{ sq. ins};$$

$$\therefore \cancel{\pi} dh = 16\cancel{\pi};$$

$$\therefore\ h=\frac{16}{d}$$

$$=\frac{16}{8}$$

$$=\underline{2''}.$$

Distance of water level from H.P. $=2''$.
Distance of centre from H.P. $=4''$.
$\therefore$ " " water level " centre $=4-2=\underline{2''}$.

$$\frac{\mathsf{CN}}{\mathsf{CB}}=\sin\alpha;$$

$$\therefore\ \sin\alpha=\tfrac{2}{4}=0{\cdot}5;$$

$$\therefore\ \underline{\alpha=\sin^{-1}0{\cdot}5=30^\circ}.$$

Distance of A from H.P. $=\mathsf{AP}$

$$=\mathsf{AM}+\mathsf{MP}=\mathsf{AM}+h.$$

Now $$\frac{\mathsf{AM}}{\mathsf{AB}}=\sin 30^\circ;$$

$$\therefore\ \mathsf{AM}=\mathsf{AB}\sin 30^\circ$$

$$=8\times 0{\cdot}5$$

$$=\underline{4''}.$$

Substituting, we obtain

$$\mathsf{AM}+h=4+2,$$

i.e. dist. of A from H.P. $=\underline{6''}$.

7. A boiler with hemispherical ends is 4′ dia. and the plates are $\frac{5}{8}''$ thick. Find the wt. of one end, neglecting overlaps and rivets. 1 sq. ft. of $\frac{1}{8}''$ plate $=5{\cdot}1$ lbs.

Superficial area of one end $=\dfrac{\pi d^2}{2}$

$$=\frac{\pi\times 4^2}{2}$$

$$=\underline{8\pi\text{ sq. ft.}}$$

Weight = area in sq. ft. $\times$ wt. of 1 sq. ft. of $\frac{5}{8}''$ plate.

$=8\pi\times 5{\cdot}1\times 5$
$=8\times 25{\cdot}5\pi$
$=204\pi$
$=\underline{640{\cdot}9\text{ lbs, say }641\text{ lbs.}}$

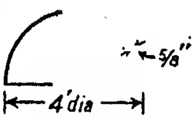


FIG. 133.

8. The sketch shows the proportions of a snap- or cup-headed rivet. Determine the volume of the head in terms of the diameter of the rivet.

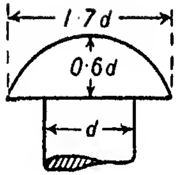


FIG. 139.

$$\left.\begin{array}{r}\text{Vol. of spherical segment,}\\ \text{i.e. rivet head,}\end{array}\right\} = \frac{\pi h}{2}r_1^2 + \frac{\pi h^3}{6}$$

$$= \frac{\pi \times 0{\cdot}6d}{2}\left(\frac{1{\cdot}7d}{2}\right)^2 + \frac{\pi(0{\cdot}6d)^3}{6}$$

$$= \pi d^3\left(\frac{\overset{0\cdot3}{\cancel{0\cdot6}} \times \overset{0\cdot7225}{\cancel{2\cdot89}}}{\underset{4}{\cancel{8}}} + \frac{\overset{0\cdot036}{\cancel{0\cdot216}}}{\cancel{6}}\right)$$

$$= \pi d^3(0{\cdot}21675 + 0{\cdot}036)$$

$$= \pi \times 0{\cdot}2528d^3$$

$$= \underline{0{\cdot}7943d^3}.$$

Hence the volume depends on the cube of the diameter. If a rivet head, diameter d, has a volume V, then one $1\frac{1}{2}d$ has a volume $\text{V} \times (1\frac{1}{2})^3 = 3\frac{3}{8}\text{V} = 3{\cdot}375\text{V}$.

Examples to be Worked Out.

1. Find the volumes and surfaces of the following spheres:

(a)	(b)	(c)	(d)
rad. 3″,	2·76 cms.,	0·023 ft.,	32·6 mms.,
dia. 5″,	1·65 cms.,	0·012″,	1·59 ft.
(e)	(f)	(g)	(h)

2. The radius of the earth is 3960 miles about. Determine its volume.

3. The globe of an arc lamp is 16″ dia. Determine its superficial area in sq. ins. and sq. cms., assuming it to be a complete sphere.

4. A brass sphere 5″ dia. is put in a cylinder partially full of water. Determine the level of the water, the dia. of the cylinder being 7″ and the water formerly 6″ deep.

5. A sphere has a volume of 897·5 cu. cms., determine its radius.

6. A sphere is immersed in water and displaces the same volume as a cube 3·67″ edge. Find the radius.

7. A sphere 4″ dia. is dropped into an inverted conical vessel whose vertical angle is 60°. Determine the distance from the centre of the

sphere to the apex, and the volume included between the apex and the sphere.

8. Determine the weights of the spheres in (5) and (6), if 1 cu. in. weighs 0·32 lb.

9. A hollow sphere is 9″ external and 4″ internal diameter, determine the volume and wt. if 1 cu. in = 0·26 lb.

10. A hollow spherical shell is 9″ external and 8″ internal dia., determine its volume.

11. A sphere 9″ dia. rests on the horizontal plane and is cut by two horizontal planes at distances of 3″ and 8″ from the point of contact. Determine the volume and curved surface of each of the three portions.

12. The radius of the plane end of a segment of a sphere is 3″ and the height 2″. Find the volume and curved surface.

13. The radii of the plane ends of a spherical zone are 1·5″ and 2·5″, and the height of the zone 3·5″. Determine the volume.

14. The radii of the plane ends of a spherical zone are 2·5″ and 3·75″ and the vol. of sphere from which it was cut is 265 cu. ins. Determine the height of the zone.

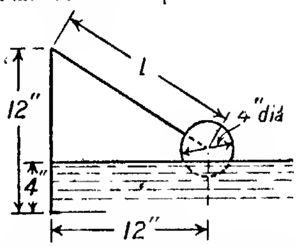
15. The radii of the plane ends of a spherical zone are 4·8 cms. and 2·6 cms. and the dia. of the sphere 15″. Determine the volume of the zone.

16. A spherical ball for a cistern is 4″ dia. Calculate the wt. of air it contains. 1 cu. in. of air = ·00004672 lb.

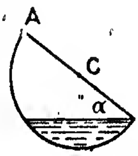
17. A sphere 12″ dia. is penetrated axially by a cylindrical hole 6″ dia. Find the volume of the remaining solid.

18. A cylinder 5″ dia. is shaped (hollowed out) to fit on a sphere 10″ dia., determine the area in contact. Draw plan and elevation to scale.

19. The arrangement is that of a cistern. Find the length of the lever l, if 25 % of the surface of the sphere is wet.

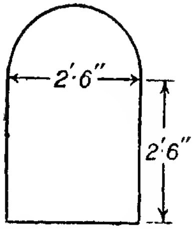


20. A hemispherical bowl 10″ dia., with $\frac{1}{4}$ of its surface wet, rests on a horizontal plane and is tilted until the water is just on the point of overflowing, as shown in the figure. Find (1) the distance of the water level from the centre; (2) α; (3) the distance of A from the horizontal plane. Draw to scale a plan and elevation.



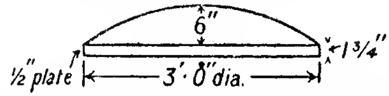
21. The end of a boiler is spherical and the plate $\frac{3}{4}''$ thick. The diameter is 6′ 9″. Determine the weight of one end. 1 sq. ft. of $\frac{1}{8}''$ steel plate = 5·1 lbs.

22. The diagram shows a dome covering for a steam engine. Find the weight if $\frac{1}{16}''$ sheet steel. 1 sq. ft. $\frac{1}{8}''$ plate = 5·1 lbs.

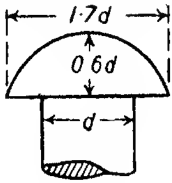


23. A cube is inscribed in a sphere 25 cms. dia., determine its edge.

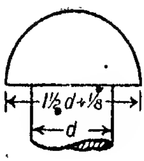
24. The following shows a dished boiler end plate. Find the weight of the plate. 1 sq. ft. of steel plate = 5·1 lbs.



25. The sketch shows the usual proportions for a snap-headed rivet. Find the wt. of a rivet head for a $\frac{3}{4}''$ dia. rivet, and hence deduce the wt. of a rivet 1″ dia. 1 cu. in. of steel = 0·28 lb.



26. Find the weight of the head of a hemispherical headed bolt when $d = 1\frac{1}{4}''$



27. A football is 11″ dia. Find the volume of air it contains.

28. If the bladder of the ball in (27) is 0·025″ thick when inflated, find its volume and wt. 1 cu in. of rubber = 0·03 lb.

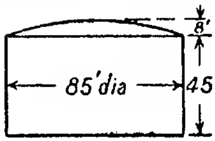
29. Show that for a given superficial area of 100 sq. ins., the volume of a sphere is greater than that of a cube.

30. The diameter of a ball at the end of a fly-press lever is 7″. Find its weight. 1 cu. in. C.I. = 0·26 lb.

31. A rectangular box 25 × 15 × 20 is packed with spheres 5″ dia. Find the number of spheres and the percentage void.

32. A hemispherical bowl is full of water. The water weighs 40 lbs. Find the radius of the bowl, if 1 cu. ft. water = 62·5 lbs.

33. The figure shows the elevation of a gas holder with a spherical shaped top. Find its cubical contents and the area of steel plate necessary for its construction.



CHAPTER IX.

TRANSFORMATION OF FORMULAE.

IF $ab=cd$, then we have $a=\frac{cd}{b}$;

for, multiplying both sides by $\frac{1}{b}$,

we get $$a\cancel{b}\times\frac{1}{\cancel{b}}=cd\times\frac{1}{b},$$

$$i.e.\quad a=\frac{cd}{b}. \quad\text{.............................(1)}$$

Similarly it can be shown that

$$b=\frac{cd}{a}, \quad\text{.............................(2)}$$

$$c=\frac{ab}{d}, \quad\text{.............................(3)}$$

$$d=\frac{ab}{c}, \quad\text{.............................(4)}$$

$$\frac{a}{c}=\frac{d}{b}, \quad\text{.............................(5)}$$

$$\frac{a}{d}=\frac{c}{b}. \quad\text{.............................(6)}$$

The original equation $ab=cd$ has been transformed six times. Moreover, there are seven different ways in which the relation between a, b, c and d has been expressed.

The above results point to a very simple rule: Quantities in the numerator after transformation are placed in the denominator, and *vice-versa*. Or, those at the top go to the

bottom, and those at the bottom go to the top after transformation.

$$\frac{ab}{1} \searrow \frac{cd}{1}, \quad \therefore \quad a = \frac{cd}{b}.$$

$$\frac{ab}{1} \swarrow \frac{cd}{1}, \quad \therefore \quad d = \frac{ab}{c}.$$

$$\frac{ab}{1} \times \frac{cd}{1}, \quad \therefore \quad \frac{a}{c} = \frac{d}{b}.$$

$$\frac{a}{d} \times \frac{c}{b}, \quad \therefore \quad \frac{ab}{1} = \frac{cd}{1},$$ and so on.

EXAMPLES.

1. Suppose we have a relation such as $xyz = rps$, and it is required that one of the six quantities shall be determined. Let this quantity be p. Draw a ring round p and eliminate all other quantities on the same side, thus:

$$\frac{xyz}{1} = \frac{rps}{1};$$

$$\therefore \; p = \frac{xyz}{rs},$$ from previous rule.

2. $$\frac{xyz}{abc} = \frac{rps}{mnq};$$ let it be required to find n.

By cross-multiplication, we get

$$xyz \times mnq = rps \times abc,$$

and $$\therefore \; n = \frac{rps \times abc}{xyz \times mq}.$$

3. Suppose $x^2 pm = n^2 q$; find x.

Then $$x^2 = \frac{n^2 q}{pm};$$

$$\therefore \; x = \sqrt[2]{\frac{n^2 q}{pm}}$$

$$= \frac{nq^{\frac{1}{2}}}{p^{\frac{1}{2}} m^{\frac{1}{2}}}$$

$$= nq^{\frac{1}{2}} p^{-\frac{1}{2}} m^{-\frac{1}{2}}.$$

4.
$$\frac{r^2m^3n}{d^3st^2}=\frac{y^2z^3}{x^3q}\text{; find } x.$$

Then
$$x^3qr^2m^3n=y^2z^3d^3st^2;$$

$$\therefore\quad x^3=\frac{y^2z^3d^3st^2}{qr^2m^3n};$$

$$\therefore\quad x=\sqrt[3]{\frac{y^2z^3d^3st^2}{qr^2m^3n}}$$

$$=\frac{y^{\frac{2}{3}}zds^{\frac{1}{3}}t^{\frac{2}{3}}}{q^{\frac{1}{3}}r^{\frac{2}{3}}mn^{\frac{1}{3}}}$$

$$=\underline{y^{\frac{2}{3}}zds^{\frac{1}{3}}t^{\frac{2}{3}}q^{-\frac{1}{3}}r^{-\frac{2}{3}}m^{-1}n^{-\frac{1}{3}}}.$$

5. The following formula is used to find the horse-power of a steam engine:

$$\text{H.P.}=\frac{2p_m\mathsf{LAN}}{33000}.$$

Given $p_m=72$, $\mathsf{N}=135$, $\mathsf{L}=3{\cdot}5$, H.P. $=180$, find the area of the cylinder A.

$$33{,}000\ \text{H.P.}=2p_m\mathsf{LAN};$$

$$\therefore\quad \mathsf{A}=\frac{33{,}000\ \text{H.P.}}{2p_m\mathsf{LN}}$$

$$=\frac{\overset{5500}{\cancel{33000}}\times\overset{\overset{4}{\cancel{12}}}{\cancel{180}}}{\cancel{2}\times\underset{18}{\cancel{72}}\times 3{\cdot}5\times\underset{\underset{3}{\cancel{9}}}{\cancel{135}}}$$

$$=\frac{\overset{\overset{550}{\cancel{1100}}}{\cancel{5500}}}{\underset{9}{\cancel{18}}\times\underset{0\cdot7}{\cancel{3\cdot5}}}$$

$$=\frac{61{\cdot}1}{0{\cdot}7}$$

$$=\underline{87{\cdot}3\text{ sq. ins.}}$$

6. Find the diameter in the previous example.

$$\mathsf{A}=\frac{\pi d^2}{4};$$

$$\therefore\ 4\mathsf{A}=\pi d^2;$$

$$\therefore\ d^2=\frac{4\mathsf{A}}{\pi};$$

$$\therefore\ d=\sqrt{\frac{4\mathsf{A}}{\pi}}$$

$$=\sqrt{\frac{4\times 87\cdot 3}{\pi}}$$

$$=\sqrt{\frac{329\cdot 2}{\pi}},$$

$$i.e.\quad d=\underline{10\cdot 54'',\text{ say }10\tfrac{1}{2}''}.$$

7. The kinetic energy of a flywheel rim is given by

$$\text{K.E.}=\frac{\mathsf{W}v^2}{2g}.$$

Find W when $g=32$, $v=60$, K.E. $=480$.

$$\mathsf{W}=\frac{2g\times\text{K.E.}}{v^2}$$

$$=\frac{\cancel{2}\times 32\times\overset{4}{\cancel{480}}}{\underset{\underset{15}{\cancel{30}}}{\cancel{3600}}}$$

$$=\frac{\overset{25\cdot 6}{\cancel{128}}}{\underset{3}{\cancel{15}}}$$

$$=\underline{8\cdot 53\text{ tons}}.$$

8. The coefficient of self-induction of a coil of wire is given by $\mathsf{L}=\dfrac{4\pi n^2\mathsf{A}}{l\times 10^9}$. If $\mathsf{A}=\pi r^2$, $r=3\cdot 5$, $\mathsf{L}=0\cdot 05$, $l=40$, find n.

$$\mathsf{L}\times l\times 10^9=4\pi n^2\mathsf{A};$$

$$\therefore\ n^2=\frac{\mathsf{L}\times l\times 10^9}{4\pi\mathsf{A}};$$

$$\therefore\ n=\sqrt{\frac{\mathrm{L}\times l\times 10^{9}}{4\pi\mathrm{A}}}$$

$$=\sqrt{\frac{0{\cdot}05\times \overset{10}{\cancel{40}}\times 10^{9}}{\cancel{4\pi}\times\cancel{\pi}\times 3{\cdot}5\times 3{\cdot}5}}$$

$$=\sqrt{\frac{0{\cdot}05\times 10^{9}}{3{\cdot}5\times 3{\cdot}5}}\qquad(\pi^{2}=10)$$

$$=\sqrt{\frac{50}{3{\cdot}5^{2}}\times 10^{6}}$$

$$=\sqrt{\frac{50}{3{\cdot}5^{2}}}\times 10^{3}$$

$$=2{\cdot}02\times 10^{3}$$

$$=\underline{\underline{2020.}}$$

9. The velocity ratio of Weston's pulley tackle is given by $\text{V.R.}=\frac{2d_1}{d_1-d_2}$. Find d_2 when V.R. $=20$, $d_1=12''$.

$$\text{V.R.}=\frac{2d_1}{d_1-d_2};$$

$$\therefore\ d_1-d_2=\frac{2d_1}{\text{V.R.}};$$

$$\therefore\ d_2=d_1-\frac{2d_1}{\text{V.R.}}$$

$$=12-\frac{\cancel{2}\times\overset{1\cdot 2}{\cancel{12}}}{\cancel{20}}$$

$$=\underline{\underline{10{\cdot}8''.}}$$

It is of interest to notice that the V.R. of this tackle should not exceed about 33. If the V.R. is in excess of 33, the load is not self-sustaining, *i.e.* the machine is reversible and the load runs back when the effort is released. The efficiency of an irreversible or self-sustaining machine cannot exceed 0·5, while a reversible one has an efficiency in excess of 0·5. The reversibility and irreversibility of machines depends on friction.

10. In the theory of torsion the following formula occurs:

$$\frac{\mathrm{G}\theta}{l}=\frac{\mathrm{T}}{\mathrm{J}},\ \text{where}\ \mathrm{J}=\frac{\pi d^{4}}{32}.$$

Find θ when $d=5$, $\mathsf{T}=200{,}000$, $l=150$, $\mathsf{G}=12\times10^6$.

$$\frac{\mathsf{G}\theta}{l}=\frac{\mathsf{T}}{\mathsf{J}};\qquad\therefore\ \theta=\frac{\mathsf{T}l}{\mathsf{GJ}}$$

$$=\frac{\overset{2}{\cancel{200{,}000}}\times\overset{5}{\cancel{150}}}{\underset{\underset{2}{4}}{\cancel{12}}\times\underset{10}{\cancel{10^6}}\times\frac{\pi\times5^4}{32}}$$

$$=\frac{\overset{16}{\cancel{32}}\times\cancel{5}}{\cancel{2}\times\pi\times\cancel{5^4}^{3}}$$

$$=\frac{16}{125\pi}$$

$=\underline{0{\cdot}041\text{ about}}$. (This is the twist in radians.)

11. The impedance of an electric circuit is

$$\mathbf{I}=\sqrt{r^2+p^2\mathsf{L}^2}.$$

Find r when $p=100\pi$, $\mathsf{L}=0{\cdot}02$, $\mathbf{I}=7{\cdot}85$.

$$\mathbf{I}=\sqrt{r^2+p^2\mathsf{L}^2};$$
$$\therefore\ \mathbf{I}^2=r^2+p^2\mathsf{L}^2;$$
$$\therefore\ r^2=\mathbf{I}^2-p^2\mathsf{L}^2;$$
$$\therefore\ r=\sqrt{\mathbf{I}^2-p^2\mathsf{L}^2}$$
$$=\sqrt{7{\cdot}85^2-(100\pi)^2\times(0{\cdot}02)^2}$$
$$=\sqrt{61{\cdot}63-10^5\times0{\cdot}0004}$$
$$=\sqrt{61{\cdot}63-40}$$
$$=\sqrt{21{\cdot}63}$$
$$=\underline{4{\cdot}65}.$$

$(100\pi)^2=100^2\times\pi^2$
$=10^4\times10$
$=\underline{10^5}.$
$\underline{(\pi^2=10\text{ approximately.})}$

The graphical representation of the above formula is given below, and is of importance to electrical engineers.

From the properties of the right-angled triangle we have $\mathbf{I}^2=r^2+p^2\mathsf{L}^2$ (see p. 23);

$$\therefore\ \mathbf{I}=\sqrt{r^2+p^2\mathsf{L}^2}.$$

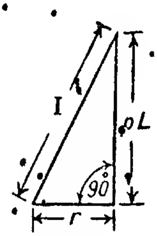


FIG. 140.

12. The combined electrical resistance of three wires in parallel is given by

$$\frac{1}{\mathsf{R}}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}.$$

Find R when $r_1=0{\cdot}56$, $r_2=1{\cdot}93$, $r_3=1{\cdot}12$.

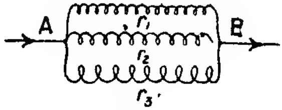


FIG. 141.

$$\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$$
$$=\frac{1}{0{\cdot}56}+\frac{1}{1{\cdot}93}+\frac{1}{1{\cdot}12}$$
$$=1{\cdot}785+0{\cdot}5181+0{\cdot}893$$
$$=3{\cdot}196;$$
$$\therefore\ R=\frac{1}{3{\cdot}196}$$
$$=0{\cdot}3129,\text{ say }0{\cdot}313.$$

Observe that the combined resistance is less than any of the three resistances, because there are three paths which the current can take in its passage from A to B.

Examples to be Worked Out.

1. Find x in each of the following cases:

(a) $\frac{x}{y}=\frac{z}{w}$. (b) $xy=wz$. (c) $\frac{y}{x}=\frac{w}{z}$.

(d) $wz=\frac{x}{y}$. (e) $xyz=abc$. (f) $pyz=xmn$.

(g) $\frac{umd}{xzq}=\frac{abc}{str}$. (h) $yzp^2=\frac{b^2cd}{x^2rs}$. (i) $\frac{z^3d^3m^3}{anq}=\frac{r^2p^9}{x^3y^4}$.

2. The power in an electric circuit is given by $P=\frac{E^2}{R}$. Find E when $P=371{,}250$ and $R=1{\cdot}459$.

3. The pressure generated by a dynamo is given by $E=4TMN$. Find M when $E=460\times10^8$, $T=75$, $N=35$.

4. In the theory of beams the following formula occurs: $\frac{M}{EI}=\frac{1}{R}$. Find I when $M=15{,}280$, $E=29\times10^6$, $R=8950$.

5. The centrifugal force on a mass m spinning in a circle of radius r is $C=\frac{mv^2}{r}$. Find v when $C=15{\cdot}93$, $r=17{\cdot}36$, $m=0{\cdot}732$.

6. In the formula $\text{H.P.}=\frac{2p_m\mathsf{LAN}}{33{,}000}$, find A the area of the engine cylinder, given $p_m=75$, $\mathsf{N}=125$, $\mathsf{L}=4$, $\text{H.P.}=265$.

7. In (6) find the diameter, if the area is in square inches.

8. Universal gravity causes an attraction between two masses m_1 and m_2, at a distance d apart. This attraction is given by $\mathsf{F}=\frac{m_1m_2}{d^2}$. Determine m_2 when $\mathsf{F}=1{\cdot}327$, $d=12{\cdot}83$, $m_1=27{\cdot}4$.

9. The volume of a sphere is $\frac{4}{3}\pi r^3$. Find the radius when the volume is 756 cubic inches.

10. The volume of a baulk of wood of square section is 12·8 cubic feet. The length is 15·9 ft.; find the side of the square in feet and in inches. ($\mathsf{V}=l\times a^2$.)

11. The velocity ratio of a screw-jack is $\text{V.R.}=\frac{2\pi r}{p}$, r being the length of the lever arm and p the pitch of the thread. Find a suitable pitch when $\text{V.R.}=200$ (about) and $r=25''$.

12. The velocity ratio of Weston's pulley tackle is $\text{V.R.}=\frac{2r_1}{r_1-r_2}$. Find r_2 when $\text{V.R.}=18$, $r_1=8{\cdot}5''$.

13. In the theory of torsion the following formula occurs:

$$\frac{\mathsf{G}\theta}{l}=\frac{\mathsf{T}}{\mathsf{J}},\ \text{where } \mathsf{J}=\frac{\pi d^4}{32} \text{ for a circular section.}$$

Find θ when $d=3$, $\mathsf{T}=50{,}000$, $l=125$, $\mathsf{G}=12\times10^6$.

14. The stress in a beam is given by $f=\frac{\mathsf{M}}{z}$. Find M when $f=7000$, $z=\frac{bd^2}{6}$, $b=2$, $d=4{\cdot}5$.

15. The formula $h=\left(\frac{\mathsf{W}+w}{w}\right)\frac{g}{\omega^2}$ is used in connection with Porter's loaded governor. Calculate h when $\mathsf{W}=25$, $w=5$, $g=32$, $\omega=2\pi n$, $n=2{\cdot}85$.

16. In connection with an electric motor the following formula occurs: $\mathsf{C}=\frac{\mathsf{E}-e}{\mathsf{R}}$. Find E when $\mathsf{C}=160$, $e=445$, $\mathsf{R}=0{\cdot}081$.

17. The kinetic energy of a flywheel rim is $\text{K.E.}=\frac{\mathsf{W}v^2}{2g}$. Determine W when $g=32$, $v=2\pi rn$, $r=3$, $n=1{\cdot}78$, $\text{K.E.}=110$.

18. The periodic time of a simple pendulum is given by $\tau=2\pi\sqrt{\frac{l}{g}}$. Find l when $\tau=2$, $g=32$.

19. The velocity of flow of water under a head h is $v=c\sqrt{2gh}$. Find h when $v=19{\cdot}7$, $g=32$ and $c=0{\cdot}62$.

20. The coefficient of self-induction of a coil of wire is given by $\mathsf{L}=\frac{4\pi \mathsf{A}n^2}{l\times10^9}$. Find n, when $\mathsf{A}=\pi r^2$, $r=3{\cdot}25$, $\mathsf{L}=0{\cdot}014$, $l=35$.

21. Prove $\frac{nR}{n-1}(T_1 - T_2) = Kp(T_1 - T_2)$ when $R = Kp - Kv$ and $n = \frac{Kp}{Kv}$.

22. Express the area of a rectangle in terms of its perimeter when the length is twice the breadth.

23. The area of a circle is πr^2; express the area in terms of the diameter. Also express the diameter in terms of the area.

24. The volume of a sphere is $\frac{4}{3}\pi r^3$, and the surface $4\pi r^2$. Express the surface in terms of the volume.

25. The perimeter of a circle is $2\pi r$, and the area πr^2; express the area in terms of the perimeter.

26. $\frac{\sin\theta}{\cos\theta} = \tan\theta$, and $\sin^2\theta + \cos^2\theta = 1$. Express (*a*) $\cos\theta$, (*b*) $\sin\theta$, in terms of $\tan\theta$.

27. The total heat of steam $H = 1082 + 0{\cdot}305t$. Given $L = 1114 - 0{\cdot}695t$, find H in terms of L.

28. The twisting moment on a solid circular shaft is given by $T = \frac{\pi f d^3}{16}$. Find d when $T = 175{,}000$, $f = 8000$.

29. The moment of inertia of a hollow circle is $I = \frac{\pi(d_1^4 - d_2^4)}{32}$. Find d_2 when $d_1 = 12{\cdot}75$, $I = 640$.

30. The kinetic energy of a rotating body is given by $\text{K.E.} = \frac{I\omega^2}{2}$. Find I when $\omega = 2\pi n$, $n = 2{\cdot}88$, $\text{K.E.} = 78{\cdot}92$.

31. The horse-power transmitted by a belt is $\text{H.P.} = \frac{(T_1 - T_2)V}{33000}$. Find V when $\text{H.P.} = 47$, $T_1 = 4T_2$ and $T_2 = 195$.

32. The total pressure on a crank pin is $P = pdl$. Find d when $p = 600$, $l = 1\frac{1}{3}d$, $P = 45{,}000$.

33. The stress in a tie bar is $f = \frac{W}{A}$, where $A = \frac{\pi d^2}{4}$. Find d when $f = 7500$, $W = 1925$.

34. The energy stored in an electric circuit is expressed by

$$W = \tfrac{1}{2} Li^2 \times 10^{-9}.$$

Find i when $W = 5{\cdot}76$, $L = 21{\cdot}6 \times 10^6$.

35. The lifting force of a magnet is $F = \frac{B^2A}{8\pi}$. Find A when $F = 489 \times 10^6$, $B = 14{,}500$.

36. The impedance of an electric circuit is $I = \sqrt{r^2 + p^2L^2}$. Find r when $p = 100\pi$, $L = 0{\cdot}01$, $I = 6{\cdot}95$.

37. The tangent of the angle of lag of an alternating-current is

$$\tan\theta = \frac{pL - \frac{1}{pK}}{r}.$$

Find K when $\tan\theta = 0{\cdot}85$, $p = 100\pi$, $L = 0{\cdot}07$, $r = 5{\cdot}76$.

38. The extension of a helical spring due to a load W is given by $x=\frac{8WD^3n}{d^4G}$. Find D when $x=0{\cdot}11$, $W=1$, $n=25$, $d=\frac{1}{4}$, $G=12\times10^6$.

39. When resonance occurs in an electric circuit, $pL=\frac{1}{pK}$. Find K when $p=2\pi n$, $n=50$, $L=0{\cdot}00056$.

40. The combined electrical resistance of three wires in parallel is given by

$$\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}.$$

Find R when $r_1=15{\cdot}3$, $r_2=12{\cdot}21$, $r_3=5{\cdot}42$.

41. The indicated horse-power of a vessel is given by $\text{I.H.P.}=kA_wV^3$. Find V when $A_w=39{,}500$, $k=\frac{1}{20000}$, $\text{I.H.P.}=6000$.

42. The pressure and volume of a perfect gas undergoing isothermal expansion or compression are related thus: $pv=c$. Find v when $c=59{\cdot}3$ and $p=14{\cdot}7$.

43. The moment of inertia of a body about an axis is given by $I=\frac{Wk^2}{g}$. Find k when $I=0{\cdot}327$, $W=42$, $g=32$.

44. The law of a machine is given by $P_f=a+bW$. Find a when $P_f=5{\cdot}75$, $b=0{\cdot}11$, $W=45$.

45. The following formula gives the diameter d of a rivet for a thickness of plate t. $d=1{\cdot}2\sqrt{t}$. Find a suitable diameter for a $\frac{7}{8}''$ plate.

MISCELLANEOUS EXAMPLES.

1. Draw a right-angled triangle $a=3{\cdot}7''$, $b=5{\cdot}9''$, and find sine, cosine and tangent of the angles A and B.

2. Draw a right angled triangle $a=2''$, $b=4''$, and by means of it, prove $\sin^2A+\cos^2A=1$, $\sin^2B+\cos^2B=1$.

3. Draw an equilateral triangle 3″ side. Drop a perpendicular from the vertex on the base. Find, (1) by drawing, (2) by calculation, the sin, cos and tan of 30° and 60°.

4. Draw an isosceles right-angled triangle, equal sides 2″ long. Find (1) by drawing, (2) by calculation, the sin, cos and tan of 45°.

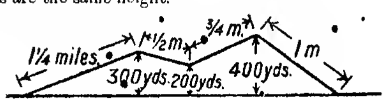
5. Construct an angle whose sine is 0·55. Measure it in degrees.

6. Construct an angle whose cos is 0·85. Measure it in degrees.

7. Construct an angle whose tan is 1·5. Measure it in degrees.

8. The shadow cast by a telegraph pole is 15·9′ long when the altitude of the sun is 50°. Find the ht. of the pole.

9. A line of telegraph poles follows a road whose plan is shown. What length per wire would have been saved if the poles had been in *one* straight line? Solve by drawing and by calculation. Assume that all the poles are the same height.



10. Two steamers leave a port at 11 a.m. and 1·45 p.m. The first sails south-east at 17 knots* and the second south-west at 25 knots. How far will they be from each other 5 hours after the departure of the first steamer? Solve by drawing and calculation.

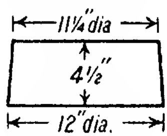
11. The pulley at the end of a crane jib is 10″ dia. How many turns will it make, and what angle will be traced out when a load is lifted 5′ 10″?

12. For every turn, the screw of a screw-jack lifts a machine ⅜″. Supposing the lever arm to be 25 ins. long, what height will the machine be lifted when the end of the lever arm has travelled 20 ft.? (The lever arm moves in a circular path 25″ rad. Strictly speaking, it moves in a helical path.)

13. The area of a fan blade has to be 25 sq. ins. and the radii 2″ and 10½″. Find the angle of the blade in degrees.

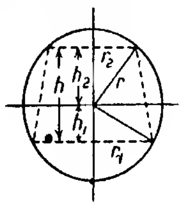
14. The commutator of an electric motor has 80 segments, each 26″ internal dia. and 31″ external dia. Determine the cross-sectional area of each segment. (The section of each segment is a portion of a ring.)

15. The clutch of a motor car is fitted with leather round its surface. Find the area of leather necessary.

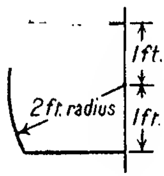


16. A circle is circumscribed about a trapezium whose parallel sides are 8″ and 5″ and altitude 6″. Find the radius, and draw to scale.

17. A sphere is circumscribed about a frustum of a cone, the radii of the ends being 15″ and 10″ and the height 8″. Find its radius, and draw a plan and elevation to scale.

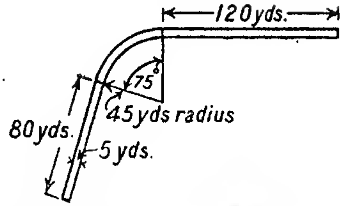


18. A barrel is formed by the revolution of an arc of a circle about a dia., as shown. Find its volume in cubic feet. Draw plan and elevation to scale.



* 1 knot = 1 nautical mile per hour
= 6080 ft. per hour.
1 naut = 1 nautical mile = 6080 ft.

19. The plan of a pavement is shown. Find what volume of cement will be necessary if it is $\frac{1}{4}''$ thick.



20. A lifebuoy is approximately elliptical in section. Major axis $=6''$, minor axis $=3''$, mean diameter $=34''$. Find its volume in cubic inches. (Multiply mean circumference by cross-sectional area.)

21. The surface of a lake is approximately elliptical, the major and minor axes being 1500′ and 1200′ respectively and the mean depth 36 ft. Find the quantity of water in gallons. 6·25 gallons $=1$ cubic foot.

22. Under a head h', the velocity of flow of water is $v=\sqrt{2gh}$ ft. per sec. Find how long it will take to drain off 5×10^6 gallons of water under an average head of 100 ft., the diameter of the supply pipe being 2′. $g=32$ ft. per sec^2.

23. The piston of a gas-engine pump has to exert a force of 7200 lbs. to compress the charge. Find its dia. if the pressure per sq. in. is 6 lbs.

24. The altitude of a conical tent is 16′ and its volume 1300 cu. ft. Find the radius of the base and the vertical angle.

25. A seam of coal has a slope of 1 in 300 for a horizontal distance of 200 yds. If the thickness at the thin end is 2′, find the thickness at the other end, and the volume of coal if the breadth of the seam is 400 ft. and the area is uniform throughout the breadth.

26. A sphere 5″ dia. is circumscribed by a cone, the radius of the base being 4″. Both rest on the H.P. Determine the semi-vertical angle and the height of the cone

27. Two toothed wheels having 120 and 75 teeth gear together. The pitch of the teeth is $1\frac{1}{4}''$. Find (1) the wheel diameters, (2) the distance between the centres.

28. Find the diameter of a circle equal in area to two circles whose diameters are 36″ and 48″ (1) by calculation, (2) graphically.

29. A right-handed helix 1″ pitch is formed on a cylinder 3″ dia. Find the angle of the helix and the length of 5 convolutions (see p. 62).

30. An isosceles triangle has its equal sides 3″ long and the contained angle 80°. Solve the triangle and find its area.

31. A steam pipe has to convey 27,000 lbs. of steam per hour to an engine at 180 lbs. per sq. inch. Find what its internal diameter must be if the velocity of flow of the steam is 6000 ft. per min. and the volume of 1 lb. $=2·49$ cu. ft. When the steam is exhausted its pressure is 3 lbs. per sq. in. and the volume of 1 lb. $=118$ cu. ft. Find the size of the exhaust pipe for a velocity of flow of 5000 ft. per min. (see p. 114).

32. The minute hand of a clock is 10″ long. Find (1) what area it sweeps out between 2.0 p.m. and 7.50 p.m., (2) the distance traversed by its extremity, (3) the angular velocity in radians per second.

33. On a map drawn to a scale of $3'' = 5$ miles, a certain district covers an area of 120 sq. ins. On another map of the same district the same area is represented by 40 sq. ins. Find the scale of the second map.

34. A surface condenser has 180 cylindrical tubes, each $\frac{5}{8}''$ external dia. and $12'$ long. Calculate its cooling surface in square feet, *i.e.* the total external surface of the tubes.

35. From a point A at sea the elevation of the top of a flagstaff is $20°$, and from a point B $\frac{1}{4}$ mile nearer land it is $30°$. Find the height of the cliff on which the flagstaff stands if the flagstaff is $80'$ high.

36. The tops of three vertical posts, whose bases are the corners of an equilateral triangle $20'$ sides, and whose heights are $10'$, $15'$ and $20'$ respectively, are joined by ropes. Find the length of each rope.

37. The cylinder of a feed-water pump is $9''$ dia. and $12''$ stroke. How many strokes will it make per min. (*i.e.* how many times must it be emptied per min.) when supplying water at the rate of 600 cu. ft. per hour? (The effective length is $12''$.)

38. A cast-iron piston ring (in the form of a hollow cylinder) is $\frac{3}{4}'' \times 1\frac{1}{2}''$ section and $36''$ external dia. Find its weight. 1 cu. in. of C.I. $= 0{\cdot}26$ lb. ($\frac{3}{4}'' =$ thickness.)

39. A pipe has a sectional area of 100 sq. ins. at one part and 75 sq. ins. at another. If 5000 cu. ft. of water flows past each section per hour, find the velocity of the water in ft. per sec. at each section.

40. In finding the radius of a curve (assumed to be circular) a chord is measured and found to be $200'$ long. The dip, *i.e.* the perpendicular distance from the centre of the arc to the chord, is $8{\cdot}5'$. Find the radius of the curve, and the angle the chord subtends at the centre.

41. A rotary air-fan delivers 6000 cu. ft. of air per min. through a rectangular discharge pipe $3{\cdot}5' \times 2'$. Find the velocity of discharge in ft. per sec.

42. A parallelogram has sides $3'' \times 2''$, and included angle $60°$. Find its area and the length of each diagonal (1) by drawing, (2) by calculation.

43. One nautical mile is the length of arc on the equator which subtends an angle of 1 minute at the centre of the earth. The radius of the earth is 3960 miles; find the number of feet in a nautical mile.

44. The distance across the flats (*i.e.* the distance between two parallel faces) of a $2''$ Whitworth hexagon nut is $3\frac{5}{32}''$. Find the distance across the corners.

45. A square and a circle have to be constructed, each equal in area to a rectangle whose perimeter is $24{\cdot}95''$. The sides of the rectangle are in the ratio $x : 2{\cdot}32x$. Find the requisite dimensions of the square and circle.

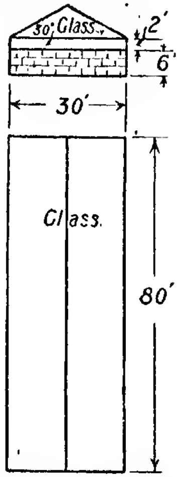
46. A portion of a $1\frac{1}{2}''$ dia. spindle is milled square, the corners of the square being on the periphery of the spindle. Find the side of the square.

47. The peripheral speed of a $\frac{1}{2}''$ twist drill is 85 ft. per min. What is its speed in r.p.m.? (This is the speed for drilling C.I.)

48. A bar $3''$ dia. and $2'\,6''$ long has to be turned with a taper of $\frac{1}{2}''$ per foot. Find (1) the dia. at the small end, (2) the angle of the cone.

49. A piece of metal used for a wedge, *e.g.* a cotter, has a taper of 1 in 19. Find the corresponding angle. (See Appendix.)

50. Find the taper corresponding to an angle of 6°. (See Appendix.)



51. The figure shows the plan and elevation of a greenhouse. Find the area of glass necessary, making no allowance for over lap on the roof. Draw the given views and a side elevation to a suitable scale.

52. The Forth Bridge is painted once in 3 years, the process being continuous. The area painted is 147 acres. If the thickness of one coat of paint is 0·005″, find the quantity required in cubic feet per annum.

53. A regular hexagon is inscribed in a circle 2″ radius. Find its area.

54. Find the angle corresponding to an incline of 1 in 96. (See Appendix.)

55. Find the incline corresponding to an angle of 50′. (See Appendix.)

56. The area of the section of water flowing over a triangular notch is 15 sq. ins., and the level of the water is 3·87″ above the vertex. Find the angle of the notch by drawing or calculation.

57. The area of the section of water flowing over a triangular notch is 14 sq. ins. and the angle of the notch is 90°. Find the level of the water above the vertex.

58. A pair of dividers has two legs each $4\frac{1}{2}''$ long. What is the angle between the legs when a distance of $3\frac{1}{2}''$ is being marked off?

59. The radius of a railway curve is 450 yds. and the angle subtended at the centre 30°. Find the total length and weight of rail necessary, assuming that it is a double line. 1 yard standard rail = 95 lbs.

60. A sphere 20″ dia. rests on a horizontal table and is cut by a horizontal plane 16″ from the table. Draw a plan and elevation showing the section. Measure or calculate the dia. of the section and find its area.

APPENDIX.

INTERPOLATION.

SUPPOSE we have a table of trigonometrical ratios without the angles subdivided into minutes, and it is necessary **to find** * **sin 27° 40′**. This can be done approximately as follows:

$$\sin 28^\circ = 0{\cdot}4695$$
$$\sin 27^\circ = 0{\cdot}4540$$
$$\text{Difference for } 60' = 0{\cdot}0155$$

$$\therefore \text{ diff. for } 40' = \frac{\overset{0\cdot00517}{\cancel{0{\cdot}0155}}}{\underset{3}{\cancel{60}}} \times \overset{2}{\cancel{40}}$$
$$= \underline{0{\cdot}0103.}$$

$$\therefore \sin 27^\circ\, 40' = 0{\cdot}4540 + 0{\cdot}0103$$
$$= \underline{0{\cdot}4643.}$$

The method of procedure is termed interpolation.

To find cos 48° 20′.

$$\cos 49^\circ = 0{\cdot}6561$$
$$\cos 48^\circ = 0{\cdot}6691$$
$$\text{diff. for } 60' = -0{\cdot}0130$$

$$\therefore \text{ diff. for } 20' = -\frac{\overset{0\cdot0043}{\cancel{0{\cdot}013}} \times \cancel{20}}{\underset{3}{\cancel{60}}};$$

$$\therefore \cos 48^\circ\, 20' = 0{\cdot}6691 - 0{\cdot}0043$$
$$= \underline{0{\cdot}6648.}$$

* 27° 40′ means 27 degrees 40 minutes. There are 60 minutes in one degree.

Notice that in this case the difference is negative. On reference to the tables, it will be seen that as the angle increases, the cosine decreases.

To find $\sin^{-1}0{\cdot}3518$, *i.e.* an angle whose $\sin = 0{\cdot}3518$.

$$\sin 21^\circ = 0{\cdot}3584$$
$$\sin 20^\circ = 0{\cdot}3420$$
$$\text{diff. for } 60' = 0{\cdot}0164$$

Now $\quad 0{\cdot}3518 - 0{\cdot}3420 = 0{\cdot}0098.$

If $\quad 0{\cdot}0164 = \text{diff. for } 60',$

then $\quad 0{\cdot}0098 = \text{diff. for } \dfrac{60}{0{\cdot}0164} \times 0{\cdot}0098$

$$= \;,, \quad ,, \quad 35{\cdot}85', \text{ say } 36'.$$

Hence $\sin^{-1}0{\cdot}3518 = 20^\circ\ 36'.$

To find $\cos^{-1}0{\cdot}7690$, *i.e.* an angle whose $\cos = 0{\cdot}7690$.

$$\cos 40^\circ = 0{\cdot}7660$$
$$\cos 39^\circ = 0{\cdot}7771$$
$$\text{diff. for } 60' = -0{\cdot}0111$$

Now $\quad 0{\cdot}7660 - 0{\cdot}7690 = -0{\cdot}003.$

If $\quad -0{\cdot}0111 = \text{diff. for } 60',$

then $\quad -0{\cdot}003 = \text{diff. for } \dfrac{60}{0{\cdot}0111} \times 0{\cdot}003$

$$= \;,, \quad ,, \quad 16{\cdot}22', \text{ say } 16',$$

so that the angle is $40^\circ - 16' = 39^\circ\ 44'$.

The same procedure applies to tangents, but the angle must not be greater than 75°. Should the angle be greater than 75°, appreciable errors are introduced. The results obtained for sines and cosines are correct to 4 decimal places in all cases. In finding inverse ratios, *e.g.* $\sin^{-1}$, $\cos^{-1}$, the angle is correct to the nearest minute.

Taper. Let ABC represent a wedge or a cone (Fig. 142). The length is x, and the breadth at the end remote from A is 1. The units in which x and 1 are measured are purely arbitrary, *i.e.* they may be inches, centimetres, feet, etc.

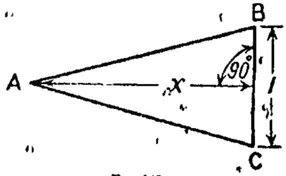


FIG. 142.

The taper is 1 in x. If x was 10, the wedge would have a taper of 1 in 10. If x was 10″, then BC would be 1″; if x was 15″, BC would be 1·5, and so on. The taper may also be written as $\frac{1}{x}$ in 1. In the present case it would be $\frac{1}{10}$ in 1 (*i.e* 1 in 10), so that it might be defined as the increase or decrease in width per inch length.

Example. A cotter is 10″ long, 2″ wide at one end and $1\frac{1}{2}$″ at the other. Find the taper and the corresponding angle.

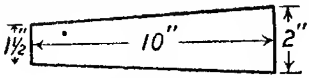


FIG. 143.

$$\text{Increase of width} = 2 - 1\tfrac{1}{2} = \tfrac{1}{2}''.$$

$$\text{Length corresponding to this increase} = 10'';$$

$$\therefore \text{ the length for an increase of } 1'' = \frac{10}{\frac{1}{2}} = 20''.$$

Hence the taper is 1 in 20.

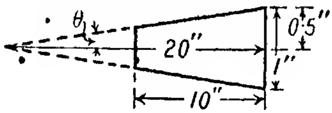


FIG. 144.

$$\frac{0\cdot5}{20} = \tan\theta;$$

$$\therefore \tan\theta = 0\cdot025;^{*}$$

$$\therefore \theta = \tan^{-1}0\cdot025 = 1^\circ\ 26'. \quad \text{(See interpolation.)}$$

Hence the angle of the cotter $= 2\theta = 2^\circ\ 52'$.

Incline. Closely allied to taper is *incline*.

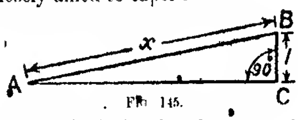


FIG. 145.

* θ may also be found as shown on p. 169.

In general, an incline is measured as 1 in x. $AB = x$ is the hypotenuse of the right-angled triangle ABC. When AB is large in comparison with BC, AC and AB are almost equal. For an incline of 1 in 10, no very serious error would be made in taking $AB = AC$. In this case, if $AB = 10''$, $AC = 9{\cdot}95''$. The difference is 0·05, *i.e.* a discrepancy of only 0·5 %. The angle corresponding to this incline is about 6°.

Approximations to Trigonometrical Ratios. If a table of ratios be consulted, it will be found that $\sin\theta$, $\tan\theta$ and θ *in radians* are approximately equal when $\theta < 12°$. Moreover, we may write

$$\underline{\sin\theta = \theta = \tan\theta \text{ when } \theta < 12°.}$$

This may be demonstrated graphically.

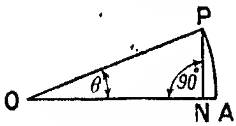


FIG. 146.

Let AP be an arc of a circle centre O, and PN a perpendicular from P on AO (Fig. 146).

Then we have $\theta = \dfrac{\text{arc}}{r}$ (see p. 68)

$$= \frac{AP}{OP}, \quad \ldots\ldots\ldots\ldots(1)$$

$$\sin\theta = \frac{PN}{OP}, \quad \ldots\ldots\ldots\ldots(2)$$

$$\tan\theta = \frac{PN}{ON}. \quad \ldots\ldots\ldots\ldots(3)$$

Now, if θ is small, $AP = PN$ approximately, and $OA = ON$ approximately.

Hence $\sin\theta = \dfrac{AP}{OP}$, (Compare with (1).)

and $\tan\theta = \dfrac{AP}{OA} = \dfrac{AP}{OP}$. (Compare with (1).)

EXAMPLES.

1. Suppose we calculate the angle corresponding to a taper of 1 in 10.

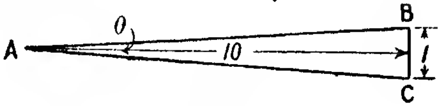


FIG. 147.

$$\theta = \frac{\text{arc}}{r} = \frac{BC}{r}, \text{ taking } BC = \text{arc drawn with A as centre,}$$

$$= \frac{1}{10}, \text{ taking } AB = 10,$$

$$= 0{\cdot}1 \text{ radian}$$

$$= 0{\cdot}1 \times 57{\cdot}3$$

$$= 5{\cdot}73^\circ.$$

Now $0{\cdot}73 \times 60 = 43{\cdot}8'$, say $44'$

Hence $\theta = 5^\circ\ 44'$, which is a fairly accurate result.

Calculating the angle corresponding to an incline of 1 in 10 would lead to the same result. In this case we have

$$\tan\theta = \frac{1}{10} = 0{\cdot}1\,;$$

but $\theta = \tan\theta$;

$$\therefore\ \theta = \frac{1^r}{10}$$

$$= \frac{57{\cdot}3}{10}$$

$$= 5{\cdot}73^\circ$$

$$= 5^\circ\ 44' \text{ (as shown above).}$$

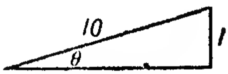


FIG. 148.

2. The angle of a wedge is 7°. Find the corresponding taper.

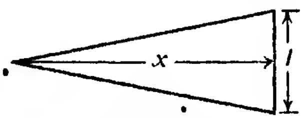


FIG. 149.

$$\theta = \frac{\text{arc}}{r} \quad (\theta = \text{radians}) \qquad = \frac{1}{x};$$

$$\therefore\ x = \frac{1}{\theta}$$

$$= \frac{1}{\frac{7}{57{\cdot}3}}$$

$$= \frac{57{\cdot}3}{7}$$

$$= 8{\cdot}19 \text{ about, } i.e. \text{ the taper is 1 in } 8{\cdot}19.$$

Logarithmic Computation. The following shows a method of setting down logarithmic working.

$$\frac{\begin{array}{c}17{\cdot}9\\ \cancel{8}\cancel{0}{\cdot}\cancel{5}\pi \times 52{\cdot}36\end{array}}{\cancel{5} \times 35{\cdot}9}$$

$$= 82{\cdot}04.$$

$$\begin{aligned}
\log 17{\cdot}9 &= 1{\cdot}2529\\
\log \pi &= 0{\cdot}4972\\
\log 52{\cdot}36 &= 1{\cdot}7190\\
&\ 3{\cdot}4691\\
\log 35{\cdot}9 &= 1{\cdot}5551\\
&\ 1{\cdot}9140
\end{aligned}$$

A.L. $1{\cdot}9140 = 82{\cdot}04.$

Cancel as much as possible before taking logarithms. Try to arrange the working so that the logarithms have to be added, *i.e.* try to make the lower line of the fraction = 1.

The following example serves to illustrate an important point in logarithmic computation.

Evaluate $\sqrt[4]{0{\cdot}0573}$.

$$\begin{aligned}
\tfrac{1}{4}\log 0{\cdot}0573 &= (\bar{2}{\cdot}7582)\tfrac{1}{4}\\
&= (-2\{-2+2\} + 0{\cdot}7582)\tfrac{1}{4}\\
&\qquad\qquad (\text{observe that } \{-2+2\} = 0)\\
&= (\bar{4} + 2{\cdot}7582)\tfrac{1}{4}\\
&= \bar{1}{\cdot}68955\\
&= \bar{1}{\cdot}6896 \text{ correct to 4 places.}
\end{aligned}$$

A.L. $\bar{1}{\cdot}6896 = 0{\cdot}4894.$

Hence $\sqrt[4]{0{\cdot}0573} = 0{\cdot}4894.$

The point to be observed is that the characteristic must be made divisible by 4. It is of interest to notice that

$$\sqrt[4]{0{\cdot}0573} > 0{\cdot}0573.$$

The n^{th} root of any number less than 1 is greater than that number, provided n is positive and greater than 1. A little consideration will show that this must always be true.

ANSWERS TO EXAMPLES.

CHAPTER I.

1. 5381 sq. ins., 304·1″. **2.** 24·14″, 18·16″. **3.** 24″, 12″.
4. 6·74 lbs. **5.** 84·7 sq. ins., 12·35″, 6·86″. **6.** 125·7 sq. ins.
7. 7200 lbs. **8.** 3510 sq. ins. or 24·36 sq. ft. **9.** 130 sq. ft.
10. 696 sq. ft. **11.** 4990 sq. yds. **12.** 6·15 miles. **13.** 5·304″.
14. 5·565″, 8·489″. **15.** 110·9 sq. ins. **16.** 76·93 sq. ins.
17. (1) 5·95 sq. ins.; (2) 5·44 sq. ins; (3) 5·3 sq. ins.; (4) 5·8 sq. ins.; (5) 3·45 sq. ins.
18. 1·73 sq. ins. **19.** 5·24 sq. ins. **20.** 4·573″.
21. 9·81 lbs **22.** 1·048″.
23. (1) Area = 1·25 sq. cm., $b = 2·01$ cms., $c = 1·41$ cm., $\hat{A} = 118°$;
(2) A = 42·3 sq. ins., $b = 9·93″$, $c = 11·14″$, $\hat{C} = 72°$;
(3) A = 41·9 sq. ft., $b = 13·24$ ft., $\hat{A} = 75°$, $\hat{C} = 29°$;
(4) A = 412 sq. yds., $a = 47·9$ yds., $b = 39·3$ yds., $\hat{C} = 26°$;
(5) A = 73·4 sq. miles, $a = 15·41$ miles, $b = 15·13$ miles, $\hat{B} = 69°$;
(6) A = 14·8 sq. cm., $\hat{A} = 30°$, $\hat{B} = 106°$, $\hat{C} = 44°$.
24. 7·8 sq. ins., 1·1 lb. **25.** 0·51″. **26.** 49 sq. ins. **27.** 78° about.
28. (*a*) 83·6 sq. cms.; (*b*) 82 sq. yds.; (*c*) 97,750 sq. yds.
29. 4·14 sq. ins. rectangle. **30.** 2·3 lbs. **31.** 28·6 sq. ins.
32. (*a*) 86·3 sq. ins.; (*b*) 116·6 sq. yds.; (*c*) 22·2 sq. mms.
33. 6·11 lbs. **34.** 823 sq. ft. **35.** 27·71 sq. ins.

CHAPTER II.

Section 1.

1. (1) 95·13 cms.; (2) 522·6 feet; (3) 2·037 yds.; (4) 3·031″.
2. 16·17″. **3.** 15·34 ft. **4.** 944·1 ft., 911·1 ft.
5. 1·436, the sum of any two sides of a triangle is greater than the third side.
6. 304·6 yds. **7.** 149·99 units, no. **8.** 30·83 yds., 182·4 yds.
9. 13·14″. **10.** 6·44 cms. **11.** 35 ft. **12.** 23·82 ft. **13.** 12 ft.

14. (1) 65·28 ft.; (2) 65 ft. and 65·9 ft. according as the 9 ft. post is above or below the 15 ft. post.
15. 5·92. 16. 35·82″, 16·84″. 17. 27·06″, 51·95″.
18. 6·13″, 12·66″, 14·14″. 19. $2\sqrt{3}=3{\cdot}464''$. 20. $a\sqrt{3}$, $\frac{\sqrt{3}}{2}$.
21. 3·35″. 22. 1·414, 1·732. 23. 2·24, 2·45.
24. 11·96 ft., 1 ft. 25. $p=3''$, $l=1{\cdot}5''$, $c=2''$, $z=2{\cdot}5''$.

SECTION 2.

1. (1) $b=21{\cdot}13$ yds., $c=24{\cdot}93$ yds., $\hat{B}=58°$;
(2) $b=13{\cdot}77$ mms., $c=25{\cdot}27$ mms., $\hat{B}=33°$;
(3) $b=75{\cdot}88$ ins., $\hat{A}=23°$, $\hat{B}=67°$; (4) $b=86{\cdot}16$ cms., $\hat{A}=40°$, $\hat{B}=50°$;
(5) $c=401{\cdot}7$ mms., $\hat{A}=51°$, $\hat{B}=39°$; (6) $c=540$ ft., $\hat{A}=13°$, $\hat{B}=77°$.
2. 95·7 ft. 3. 6400 ft. 4. 101·8 sq. ins. 5. 119·3 sq. ins., 0·854 : 1.
6. $na^2 \sin\frac{180°}{n}\cdot\cos\frac{180°}{n}$. 7. $na^2 \tan\frac{180°}{n}$, $\cos^2\frac{180°}{n}$.
8. 4·64″. 9. 9·417″. 10. 56·3 sq. ins. 11. 833 ft.
12. 17° about. 13. 1·54, 6·16, 13·87, 24·65, 38·51 sq. cms.
14. 62·61 sq. mms., 0·6261 sq. cm. 15. 85° about. 6·022.
16. 1·75 nauts. 17. 162·7 sq. ins.
18. BC = DE = 11·33 ft., AB = DF = 17·63 ft.
19. BC = 14·44 ft., AB = CD = 13 ft., $\hat{ABC}=170°$.
20. $a=12{\cdot}5°$ about, AD = 11·0 ft., DC = 0·52 ft. 21. 11·2 ft.
22. 9·166″. 23. 32·98″. 24. 64·89 mms. 25. 86·58 sq. miles.
26. True area = 3·88 sq. ins.; area of plan = 1·64 sq. ins.; true length of $a'b'=3{\cdot}07''$.
27. 20·79 ft. 28. $\frac{a}{\sqrt{3}}$, $\frac{2a}{\sqrt{3}}$, 1 : 4.
29. 25·77 sq. ins., 19·97 sq. ins., 1·29 : 1. 30. 440 sq. ft.
31. 67·5 ft. 32. 3″, 7·5″. 33. 29 ft., 23 ft., 7·76 ft., 19·28 ft.
34. $\hat{A}=53°$, $\hat{B}=37°$, $\hat{C}=90°$, $a=48$ ft., $b=36$ ft., $c=60$ ft.
35. 114·9 lbs., 96·42 lbs. 36. $Oa=4{\cdot}05''$, $\theta=18°\ 25'$, 9·6 : 1.
37. 62·8 sq. ins. 38. 4·75″.

CHAPTER III.

SECTION 1.

1. (a) 32·59 cms., 84·54 sq. cms.; (b) 302·8″, 7295 sq. ins.; (c) 264·7 yds., 5575 sq. yds.; (d) 29·06″, 67·2 sq. ins.; (e) 79·33 ft., 500·7 sq. ft.; (f) 314·2 metres, 7854 sq. metres.
2. 2·546 revs. 3. 298·7 sq. ins., 58,240 lbs. 4. 671·4 lbs.

5. $\frac{7}{8}''$. **6.** 8·3 sq. ins. **7.** 720 revs. per mile, 168 r.p.m. **8.** 3535 ft. per min. **9.** 1200 lbs. **10.** 11·04 sq. ins. **11.** 182·7 sq. ins. **12.** 8 coils, 8″. **13.** $3\pi = 9{\cdot}425$ ft. **14.** 86. **15.** 1·886″. **16.** $20\frac{1}{8}''$. **17.** 233·3 ft. **18.** 7′ 7″. **19.** 371 r.p.m., 74 r.p.m. **20.** 4·162″, 12·488″. **21.** 38″, 11·59″. **22.** 127 3″. **23.** 630,000 lbs. **24.** 9″. **25.** 25,450 lbs. **26.** 0·77″. **27.** 17·28 miles per min. **28.** 5·49 sq. ins. **29.** 14 ft. about **30.** 237 r.p.m. **31.** $31\frac{1}{2}''$ about. **32.** 9·231″, 3·820″, 15·278″, 11·777″. **33.** 1·69 : 1. **34.** 21·75 miles per hour. **35.** 55·7″. **36.** 3·89″, $\frac{\pi}{2} = 1{\cdot}571$. **37.** 5·3″, $\frac{4}{\pi} = \frac{1{\cdot}273}{1}$. **38.** 6·74 mms., 9·24 mms. **39.** $30\frac{1}{4}''$. **40.** 900 ft. per min. **41.** 1120 ft. per min. **42.** 50 sq. ins., $2r^2$. **43.** 76, 19 ft. **44.** 20″, 12″. **45.** 37 7 sq. ins. **46.** 4·86″. **48.** 32″. **49.** 4·43″. **50.** 5·64″.

Section 2.

1. (*a*) 0·655^c; (*b*) 2 374^c; (*c*) 5 627^c; (*d*) 32 09°; (*e*) 75 65°; (*f*) 310°; (*g*) 167·8°; (*h*) 235 2°; (*i*) 168 2°; (*j*) 582·4′; (*k*) 5724 cms.; (*l*) 125·1 Km.; (*m*) 417 8 cms.; (*n*) 84·89 ins; (*o*) 7·49 yds.

2. $4'\,2\frac{1}{2}''$. **3.** 0·538^c, 31°. **4.** $Oa = 2{\cdot}77''$, $\theta = 31°$, $OM = 31\frac{1}{2}''$.

5. (*a*) 59′ 4″; (*b*) 59′ 10″.

6. (*a*) 981 sq. ins.; (*b*) 25,300 sq. cms.; (*c*) 16,400 sq. yds.

7. 12·41″. **8.** 133°. **9.** 348 sq ft. **10.** 1 sq. in. **11.** 0·455 sq. ins. **12.** 8″. **13.** 8 38 rad. per sec., 20 93 ft. per sec. **14.** 97·41 cms., 0·9741 mms. **15.** 28·7°. **16.** 35·3°. **17.** 41 sq. cms. **18.** 0·417 sq. ft. **19.** 117 sq. ins. **20.** 3 613 sq. ins. **21.** 53·33 sq. cms., 0 5333 sq. dm. **22.** 0·4 sq. m. **23.** 26·18 rad. per sec. **24.** 7·89 rad. per sec. **25.** $1'\,1\frac{9}{16}''$. **26.** $0{\cdot}1612a^2$.

27. (1) 4·189″; (2) 9·425 sq. ins.; (3) 2″ rad., 12·57 sq ins; (4) 1 : 2

Section 3.

1. (*a*) $l = 37{\cdot}8''$, $r = 34{\cdot}9''$; (*b*) $l = 10{\cdot}6$ cms., $r = 6{\cdot}31$ cms.; (*c*) $l = 93{\cdot}5$ mms., $r = 112\ 8$ mms.; (*d*) $l = 8{\cdot}49'$, $r = 4\ 78'$.

2. (*a*) $A = 57{\cdot}3$ sq. ins., $\theta = 128°$; (*b*) $A = 592$ sq. cms., $\theta = 175°$; (*c*) $A = 22{\cdot}1$ sq. ft., $\theta = 169°$.

3. 11 sq. ft. **4.** Steam space = 12·25 sq. ft., water space = 21·5 sq. ft.

5. 2020 lbs., 2880 lbs., 2020 lbs. **6.** 0·8 %.

7. $c = 80{\cdot}6$ mms., $h = 16{\cdot}12$ mms., $r = 13{\cdot}16$ mms.

8. 351 sq. ft. **9.** 43·74 sq. ins., 6·52 sq. ins.

CHAPTER IV.

1. (a) 78·4″, 446·6 sq. ins.; (b) 59·55 cms., 252·8 sq. cms.; (c) 2509 mms., 500,000 sq. mms.; (d) 22·8′, 34·61 sq. ft.
2. 10·02″, 7·59″; perimeter = 27·67″.
3. 37·7 sq. ins., 7″ dia., 38·48 sq. ins.
4. 27·17″, 21·74″; area = 1857 sq. ins. 5. 1·117″. 6. 2·44″.
7. 18 lbs. 8. 11·26″. 9. 0·642″. 10. 14·14″.

CHAPTER V.

1. 392·2 cu. ins. 2. 9·91″. 3. 16·39. 4. 21·3″.
5. 21·57″. 6. 62·75 cms. 7. 321·5 sq. ins., 31,540 sq. cms.
8. 12·58″. 10. 0·434 lbs. per sq. in. 11. 406·3″, 33·86′.
12. 484 lbs. 13. (1) 125 : 8; (2) 25 : 4. 14. 52 %.
15. 48 units of length. 16. $\frac{1}{10}$ unit of length 17. 13·74″.
18. 4″, 12″, 16″. 19. 85·5 cu. ins., 132 sq. ins. 20. 6·74 cms.
21. 10·54″. 22. 3·94″. 23. 6·02″, 9·03″, 15·05″.
24. 1·14 lb. 25. 19″. 26 508 lbs. 27. 114·3 cu. ins.
28. 1036 cu. ins. 29. 1·02″ sq. 30. 66·7 lbs. 31. 896,000 cu. ft.
32. 0·28 lb. 33. 28,350 cu. cms. 34. 11·72′ from bottom.
35. 14·7 lbs. per sq. in. 36 16 cu. ins. 37. 0·0837 cu. in., 0·0234 lb.
38. 13·28 lbs. 39. 0·704″. 40. 49·1 lbs. 41. 5 cu. ins., 1·4 lb

CHAPTER VI.

1. (a) 369,500 cu. ins., 20,380 sq. ins.; (b) 169,500 cu. ins., 8350 sq. ins.; (c) 857·2 cu. ft., 371·5 sq. ft; (d) 1·88 cu. ins., 6·09 sq. ins.; (e) 0·0000797 cu. in., 0·00245 sq. in.; (f) 4·68 cu. ins., 4·05 sq. ins.; (g) 6·26 cu. cms., 45·37 sq. cms.; (h) 89·95 cu. ft., 143·9 sq. ft.; (i) 619·1 cu. ins., 315·1 sq. ins.; (j) 0·2 cu. in., 3·15 sq. ins.
2. 0·371 cm. 3. 3·43″. 4. $r=3·438″$, $h=6·36″$.
5. $r=8·766′$, $h=14·94′$. 6. 16·34 sq. ft., 31·42 sq. ft.
7. 1·904 lb. per stroke, 18,300 lbs. per hour. 8. 6 %.
9. 23,130 lbs., or 10·32 tons. 10. 95 sq. dm. 11. $76\frac{3}{8}$″.
12. 2480 lbs. 13. 1934 lbs. 14 10,340 lbs., 15,495 lbs.
15. 7600 lbs. 16. 21·46 %. 17. 21·46 %. 18. 2·94″.
19. 1000 sq. ft. 20. 6440 cu. ft. per hour.
21. 283,000 cu. ft. per min. 22. $a=0·945r$, $b=0·529r$.
23. 16,130 ft. 24. 2′ $9\frac{3}{8}$″.
25. L.P.C. = 12·88 cu. ft., H.P.C. = 4·29 cu. ft., dia. of L.P.C. = 26″.
26. 8580 lbs. per hour, 440 H.P. 27. 17,240 cu. mms.

28. 20″, 32″, 9 : 26 : 64 or 1 : 2·8 : 7.

29. Tubes = 1242 sq. ft., total = 1401·5 sq. ft., $\frac{H}{G} = 68 \cdot 4$.

30. 30·4 cu. ins., 7·9 lbs. **31.** 0·643″. **32.** 453 lbs.

33. 22·23 lbs. **34.** 198·8 cu. ins. **35.** $d = 5 \cdot 47''$, $h = 3 \cdot 87''$.

36. 0·246 cm. **37.** 628 lbs. **38.** 0·197″. **39.** 27·09 %.

40. 9·42 sq. ins. **41.** 1·128. **42.** 1·25 lb. **43.** $3\frac{1}{2}''$.

44. 23·2 sq. ft., 4·1 ft. per sec.

45. 3·308″ above and below the plane containing the axes.

CHAPTER VII.

1. (*a*) P.S. = 33·18 sq. ins., C.S. = 69·7 sq. ins., $v = 66 \cdot 39$ cu. ins.

(*b*) P.S. = 26,880 sq. ins., C.S. = 36,500 sq. ins., $v = 761{,}500$ cu. ins.

(*c*) P.S. = 0·049 sq. ft., C.S. = 0·885 sq. ft., $v = 0 \cdot 0368$ cu. ft.

(*d*) P.S. = 30·69 sq. yds., C.S. = 96, $v = 95 \cdot 1$ cu. ft.

2. 471·4 cu. ins. **3.** $r = 6 \cdot 68''$, $h = 18 \cdot 36''$. **4.** $r = 2 \cdot 528''$, $h = 3 \cdot 792''$

5. $m_1 = 8 \cdot 98''$, $m_2 = 3 \cdot 59''$, $h = 9 \cdot 66''$. **6.** $h = 2 \cdot 12''$.

7. $l = 17 \cdot 76''$, $\theta = 3 \cdot 954°$ or $226 \cdot 5°$. **8.** $\theta = 22°$, $h = 13 \cdot 91'$.

9. 3·09″ from base. **10.** 1·91″ from base, 4·6″ from base.

11. 4·39″ from base. **12.** 4·1″ from base.

14. Altitude = 6·93″, C.S. = 100·6 sq. ins., $\theta = 180°$. **15.** 3·17″, 18·25″.

16. 224·8 lbs. **17.** 2·09 lbs. **18.** $v = 48 \cdot 13$ cu. ins., C.S. = 217 sq. ins.

19. $r_1 = 2 \cdot 441''$, $r_2 = 1 \cdot 326''$. **20.** 330 sq. ins.

21. 50 cu. ins., 74·5 sq. ins. **22.** 0·073 lb. **23.** 6·68″.

24. V = 279 cu. ins., C.S. = 279 cu. ins.

25. V = 4290 cu. ins., A = 1040 sq. ins. **26.** 3300 cu. ins.

28. 387 lbs. **29.** (*a*) 21·24 lbs ; (*b*) 198,000 lbs. **30.** 151 lbs.

31. 7·958″, 11·141″, 71° 4′, 108° 56′. **32.** 76·2 cu. ins.

CHAPTER VIII.

1. (*a*) $v = 113 \cdot 1$ cu. ins., $s = 113 \cdot 1$ sq. ins. ;

(*b*) 88·04 cu. cms., 95·72 sq. cms. ;

(*c*) $5 \cdot 1 \times 10^{-5}$ cu. ft., $6 \cdot 65 \times 10^{-3}$ sq. ft.

(*d*) 145,100 cu. mms., 13,360 sq. mms.

(*e*) 65·45 cu. ins., 78·54 sq. ins. ; (*f*) 2·353 cu. ins., 8·555 sq. ins. ;

(*g*) $9 \cdot 05 \times 10^{-7}$ cu. in., $4 \cdot 53 \times 10^{-4}$ sq. in. ;

(*h*) 2·105 cu. ft., 7·943 sq. ft.

2. 26×10^{-10} cu. miles. **3.** 804 sq. ins., 5190 sq. cms. **4.** 7·7″.

5. 5·98 cms. **6.** 2·277″. **7.** 4″, 4·19 cu. ins.

8. (5) 17·53 lbs. ; (6) 15·82 lbs. **9.** 348 cu. ins., 90·5 lbs.

10. 114 cu. ins.
11. 98·97 cu. ins., 259·6 cu. ins., 13·09 cu. ins., 84·82 sq. ins., 141·4 sq. ins., 28·27 sq. ins.
12. 32·45 cu. ins., 40·84 sq. ins. **13.** 69·17 cu. ins. **14.** 4·45″.
15. 1697 cu. ins. **16.** 0·00157 lb. **17.** 587 cu. ins. **18.** 21 sq. ins.
19. 13·9″. **20.** 3·75″, 49° about, 8·75″. **21.** 2190 lbs.
22. 75·1 lbs. **23.** 14·43 cms. **24.** 188 lbs.
25. 0·094 lb., $\frac{64}{27} \times 0·094 = 0·222$ lb. **26.** 0·586 lb. **27.** 697 cu. ins.
28. 9·5 cu. ins., 0·29 lb. **29.** Sphere = 94·1 cu. ins., cube = 68·1 cu. ins.
30. 46·7 lbs. **31.** 60 spheres, 47·64 %. **32.** 7·98″.
33. 278,300 cu. ft., 17,880 sq. ft.

CHAPTER IX.

1. (a) $\frac{y^4}{w}$; (b) $\frac{wz}{y}$; (c) $\frac{yz}{w}$; (d) wyz; (e) $\frac{abc}{yz}$; (f) $\frac{pyz}{mn}$;
(g) $\frac{nmdstr}{abczq}$; (h) $\sqrt{\frac{b^2cd}{rsyzp^2}}$; (i) $\sqrt[3]{\frac{r^2p^3anq}{z^3d^3m^3y^4}}$.
2. 736. **3.** $4·38 \times 10^6$. **4.** 4·526. **5.** 19·44. **6.** 117.
7. $12\frac{1}{4}$″. **8.** 7·971. **9.** 5·65. **10.** 0·897′, 10·77″. **11.** $\frac{13}{16}$″.
12. 7·55. **13.** 0·0655. **14.** 47,250. **15.** 0·6. **16.** 458.
17. 6·22. **18.** 3·24. **19.** 15·8. **20.** 1076.
22. $A = \frac{P^2}{18}$. **23.** $A = \frac{\pi d^2}{4}$, $d = 2\sqrt{\frac{A}{\pi}}$. **24.** $s = (3V)^{\frac{2}{3}}(4\pi)^{\frac{1}{3}}$.
25. $A = \frac{P^2}{4\pi}$. **26.** $\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}}$, $\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$.
27. $H = 1570 - 0·439L$. **28.** $4\frac{13}{18}$ about. **29.** 11·69.
30. 0·492. **31.** 2650. **32** $7\frac{1}{16}$ about. **33.** $\frac{9}{16}$ about.
34. 23·1. **35.** 58·45. **36.** 6·19. **37.** 0·000183.
38. 2·95. **39.** 0·0179. **40.** 3·015. **41.** 14·5.
42. 4·034. **43.** 0·5. **44.** 0·8. **45.** $1\frac{1}{8}$″.

MISCELLANEOUS EXAMPLES.

1. sin A = 0·53, cos A = 0·86, tan A = 0·63, sin B = 0·86, cos B = 0·53, tan B = 1·6.
3. See tables. **4.** See tables. **5.** 35° about. **6.** 32° about.
7. 56° about. **8.** 19 ft. **9.** 88 yds. **10.** 102 nauts.
11. 2·23 revs., 802°. **12.** 0·57″. **13.** 27°. **14.** 2·8 sq. ins.
15. 165 sq. ins. **16.** 4·56″. **17.** 15·48″. **18.** 23 cu. ft.
19. 246 cu. ft. **20.** 1530 cu. ins. **21.** $3·18 \times 10^8$ gallons.
22. 53 min. 3 sec. **23.** 3′ 3″ dia. **24.** 8′ 10″, 58°.
25. 4 ft., 72×10^7 cu. ft. **26.** 26°, 8·2″.

27. 47·745″, 29·841″ diameters, 38·793″ centres. **28** 60″.

29. 6°, 47·4″. **30.** 4·43 sq. ins., $\hat{B}=\hat{C}=50°$, $a=3·86″$. **31.** $5\frac{7}{8}″$, $3′\,8\frac{1}{8}″$.

32. 183·2 sq. ins., 366·4 ins., $\frac{\pi}{1800}=0·00175$ rad. per sec.

33. $1″=\frac{5}{\sqrt{3}}=2·887$ miles. **34.** 495 sq. ft. **35.** 1225 ft.

36. 20·62 ft., 20·62 ft., 22·36 ft. **37.** 22·63. **38.** 32·4 lbs.

39. 2 ft. per sec., 2·67 ft. per sec. **40.** 592 ft., 19°.

41. 14·3 ft. per sec. **42.** 5·2 sq. ins., 2·65″, 4·35″.

43. 6080 ft. **44.** 3·645″. **45.** 5·72″ side, 6·46″ dia.

46 1·06″. **47.** 649 r.p.m. **48.** 1·917″, 2° 23′. **49.** 3°.

50. 1 in 9·54. **51.** 3471 sq. ft. **52.** 890 cu. ft. **53.** 10·39 sq. ins

54. 36′. **55.** 1 in 69. **56.** 90°. **57.** 3·74″. **58.** 45° 47′

59. 943 yds., 89,580 lbs. or about 40 tons. **60.** 16″ dia., 201 sq. ins.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0126	0170						4	9	13	17	21	26	30	34	38
						0212	0263	0294	0334	0374	4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0559						4	8	12	16	19	23	27	31	35
						0607	0645	0662	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0626	0854	0899	0934	0969					3	7	11	14	18	21	25	28	32
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	28
					1561	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1816	1847	1875	1903					3	6	9	11	14	17	20	23	[illegible]
							1931	1959	1987	2014	3	5	8	11	14	16	19	22	25
16	2041	2006	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2105	2430					3	5	8	10	13	15	18	20	23
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2646						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2768	2810	2833	2656	2678						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3315	3365	3385	3404	2	4	6	8	10	12	14	16	16
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	16
27	4314	4330	4316	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4516	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4600	4814	4829	4613	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4912	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5169	5172	1	3	4	6	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	0	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	2	4	5	6	6	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	0	7	6	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	6	9	10
38	5798	5609	5621	5632	5643	5855	5866	5877	5886	5899	1	2	3	5	6	7	6	9	10
39	5911	5922	5933	5014	5955	5066	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	5042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6126	6138	6149	5160	6170	5180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	8253	6263	6274	6284	6294	5304	6314	6325	1	2	3	4	5	6	7	6	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	5454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	5542	6551	6561	6571	6580	5590	6599	6609	6616	1	2	[illegible]	4	5	0	7	8	9
46	6626	6637	6646	6656	6665	6675	6664	6693	5702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	5776	6785	6794	6603	1	2	3	4	5	5	6	7	6
48	6612	6821	6830	5839	6646	6857	6866	6875	6684	5893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	[illegible]	4	4	5	6	7	8
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52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	[illegible]	[illegible]	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
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78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

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·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
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·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
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·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
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·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
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·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
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·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
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·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
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·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
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·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
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·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
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·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
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·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
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·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
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·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
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·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
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·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
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·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

Angle		Chord.	Sine.	Tangent.	Co-tangent.	Cosine			
Degrees	Radians.								
0°	0	000	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
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33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine.	Co-tangent.	Tangent.	Sine.	Chord.	Radians.	Degrees.
								Angle.	

INDEX

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